J. Mountain Res. P-ISSN: 0974-3030, E-ISSN: 2582-5011 DOI: https://doi.org/10.51220/jmr.v21-i1.11
Vol. 21(1), (2026) 93-103

Farey Graceful Labeling of Some Complex Structures

Vanshika Tyagi[1*] • Rajeev Kumar[2] • Suraj Tyagi[3] • Ajay Kumar[4]

[1,2]*Mathematics Department, Vikrant University, Gwalior, Madhya Pradesh, India*
[3]*School of Computer Applications, Vivek University, Bijnor, Uttar Pradesh, India*
[4]*Department of Mathematics, Shaheed Srimati Hansa Dhanai Rajkiya Mahavidyalaya, Agrora (Dharmandal), Tehri Garhwal, Uttarakhand, India*

*Corresponding Author Email Id: vanshikatyagi485@gmail.com

Received: 5.4.2026; Revised: 14.5.2026; Accepted: 20.5.2026

Abstract: Graph labeling is a significant area of graph theory due to its wide range of theoretical and practical applications. Recently, Kumar et al. introduced the concept of Farey graceful labeling based on the Farey sequence, which consists of reduced rational numbers between 0 and 1 arranged in increasing order. In Farey graceful labeling, vertices of a graph are assigned distinct terms of the Farey sequence such that the induced edge labels satisfy prescribed conditions. This labeling scheme was shown to exist for several classes of graphs, including paths, cycles, hairy cycles, caterpillar graphs and super caterpillar graph. In this paper, we extend the study of Farey graceful labeling to additional families of graphs. Specifically, we prove that the W graphs, double wheel graphs DW_l, graph obtained by switching of vertices of cycle, admit Farey graceful labeling. The results presented in this work contribute to the growing literature on rational graph labeling and open new directions for further investigation of Farey-based labeling techniques.

Keywords: Graph labeling • Farey graceful labeling • Double wheel graphs • Vertex Switching.
Subject Classification (AMS): 05C78

Introduction

Graph labeling has been an important and interesting topic in graph theory for many years, mainly because of its theoretical importance as well as its practical usefulness. In general, a graph labeling means assigning numbers or other mathematical objects to the vertices or edges of a graph in such a way that certain conditions are satisfied. Many different types of graphs labelings have been introduced over time and a large amount of literature is now available on this subject in Gallian (2023). One of the earliest and most well-known labeling schemes is graceful labeling, which was introduced by Rosa (1967). After this, several new variants such as harmonious, cordial and arithmetic labelings were studied by different researchers.

Wheel graphs and their related structures form an important class of graphs because of their simple definition and symmetric structure. A wheel graph W^l is obtained by connecting a central vertex to all the vertices of a cycle. A double wheel graph is a natural extension of this idea, where a single hub vertex is joined to two disjoint cycles. These graphs appear frequently in network design and other applied areas and therefore their labeling properties have attracted attention from many authors Harary (1988), Acharya (1983). Several researchers have studied graceful and other labelings of wheel-related graphs and obtained interesting results, Bloom and Golomb (1977), Yilmaz and Cahit (2010).

In recent years, researchers have started using ideas from number theory to define new graph labeling schemes. One such labeling is Farey graceful labeling, which was introduced by Kumar et al. (2024). Based on this labeling further interesting work was carried out by Kumar et al. (2024), Kumar et al. (2024), Kumar et al. (2024), Kumar et al. (2024), Kumar et al. (2025). This labeling is based on the Farey sequence, which consists of reduced fractions between 0 and 1 arranged in increasing order. In Farey graceful labeling, distinct Farey fractions are assigned to the vertices of a graph and edge labels are obtained from the absolute differences of these fractions. This approach creates a nice connection between graph theory and classical number theory.

Another important aspect in graph theory is the study of graph operations and transformations. Vertex switching is one such operation in which the adjacency relations of a selected vertex are altered. This operation often changes the structure of the graph and it may also affect the labeling properties. Some work has already been done on the effect of switching on different graph labelings Seoud and Salim (2007), but many questions still remain open.

Although Farey graceful labeling has been defined and studied for some basic graph classes, its behaviour on wheel graphs, double wheel graphs and graphs obtained through vertex switching has not been studied in detail. This gap in the literature provides the motivation for the present work. In this paper, we attempt to study Farey graceful labeling of double wheel graphs, W graphs and graph obtained by switching of vertices of cycle. The results obtained in this work add to the existing knowledge of Farey graceful labeling and may be helpful for further research in number-theoretic graph labelings.

To enhance the understanding of the results presented in this paper, the following relevant definitions are provided.

Definition 1.1. A Farey sequence, Burton (2010) F_n of order n, for $n \in N$, is defined by the set of numbers $\frac{l}{m} \in Q$ with $0 \leq l \leq m \leq n$ and l, m are co-prime placed in increasing order.

Definition 1.2. If an injective function χ is defined on G with number of edges m such that $\chi : V \rightarrow F_{m+1}$, where F_{m+1} represents the Farey sequence of order $m+1$ and bijection $\chi^* : E \rightarrow \{1,2,3,\ldots,m\}$ defined by $\chi^*\left(\frac{c_1}{d_1}, \frac{c_2}{d_2}\right) = |c_1 d_2 - c_2 d_1|$ is induced, then labeling of graph is called Farey graceful labeling, Kumar A (2024).

Definition 1.3. A double wheel graph, denoted by DW_l, is a graph obtained from two cycle graphs C_l together with a single central vertex K_1. Represented as $DW_l = 2C_l + K_1$. The graph is constructed by taking two cycles, each of length l and adding one common central vertex. This central vertex is then connected to every vertex of both cycles. Thus, the resulting graph consists of two concentric wheel-like structures sharing the same hub and hence it is called a double wheel graph, Zhang (2015).

Definition 1.4. The W-graph, denoted by W^d, is a graph constructed using two ladder graphs L_{n+1}, where $n \geq 1$, together with a cycle graph C_3. To construct W^d, first take two copies of the ladder graph L_{n+1}. Let q_1 and x_1 be the end vertices of the two ladders. Join these two vertices by an edge. Then add a new vertex and connect it to q_1 and x_1. As a result, the vertices q_1, x_1 and the new vertex form a triangle which is a cycle graph C_3. Hence, the graph obtained by combining the two ladder graphs through this triangular connection is called the W-graph W^d.

Definition 1.5. Vertex switching, Ellingham (1991) is a graph operation in which the adjacency of a chosen vertex is modified. When this operation is performed at a vertex v, all edges incident to v are first deleted. After this, new edges are introduced between v and those vertices that were not adjacent to it earlier. In this way, the neighbourhood of the vertex v is completely altered while the rest of the graph remains unchanged.

In the following section, we will discuss the applicability of this labeling to several classes of graphs.

2. Results

According to the Definition 1.2, the subsequent findings are obtained.

Theorem 2.1. All double wheel graphs DW_l are Farey graceful for $l\geq 4$.

Proof. Clearly, $|E(DW_l)|=4l=m$, let p_r be the r^{th} vertex of the inner cycle of the double wheel graph, q_r be the r^{th} vertex of the outer cycle and z be the central vertex. The proposed labeling scheme considers two scenarios depending on whether l is odd or even. Define: $\chi: V(DW_l)\rightarrow F_{m+1}$ by,

Case 1: When l is even, $l\geq 4$

$$\chi(z)=\frac{0}{1}$$

$$\chi(p_r)=\begin{cases}\frac{r}{r+1}, & \text{if } r \text{ is odd}, 1\leq r\leq l-3\\ \frac{4l-3-r}{4l-2-r}, & \text{if } r \text{ is even}, 2\leq r\leq l-2\\ \frac{4l}{4l+1}, & \text{if } r=l-1\\ \frac{4l-2}{4l-1}, & \text{if } r=l\end{cases}$$

$$\chi(q_r)=\begin{cases}\frac{4l-4}{4l-3}, & \text{if } r=1\\ \frac{l+r}{l+r+1}, & \text{if } r \text{ is odd}, 3\leq r\leq l-3\\ \frac{3l-1-r}{3l-r}, & \text{if } r \text{ is even}, 2\leq r\leq l-2\\ \frac{4l-1}{4l}, & \text{if } r=l-1\\ \frac{2l}{2l+1}, & \text{if } r=l\end{cases}$$

Case 2: When l is odd, $l\geq 5$

$$\chi(z)=\frac{0}{1}$$

$$\chi(p_r)=\begin{cases}\frac{r}{r+1}, & \text{if } r \text{ is odd}, 1\leq r\leq l-2\\ \frac{4l+1-r}{4l+2-r}, & \text{if } r \text{ is even}, 2\leq r\leq l-3\\ \frac{2l}{2l+1}, & \text{if } r=l-1\\ \frac{2l+4}{2l+5}, & \text{if } r=l\end{cases}$$

J. Mountain Res. P-ISSN: 0974-3030, E-ISSN: 2582-5011 DOI: https://doi.org/10.51220/jmr.v21-i1.11
Vol. 21(1), (2026) 93-103

$$\chi(q_r)=\begin{cases}\frac{4l}{4l+1}, & \text{if } r=1\\ \frac{2l+2+r}{2l+3+r}, & \text{if } r \text{ is odd}, 3\le r\le l-2\\ \frac{2l+1-r}{2l+2-r}, & \text{if } r \text{ is even}, 2\le r\le l-3\\ \frac{3l+2}{3l+3}, & \text{if } r=l-1\\ \frac{2l+2}{2l+3}, & \text{if } r=l\end{cases}$$

The function χ of vertices induces the bijective function $\chi^* : E(DW_l)\rightarrow\{1,2,3,\ldots,m\}$ of edges defined by $\chi^*\left(\frac{c_1}{d_1},\frac{c_2}{d_2}\right)=|c_1d_2-c_2d_1|$. In Figure 1, the graph with $l=16(even)$, DW_{16} is labeled using the function χ with vertex labels as shown in the figure, the induced edge labels for the edge set $\{zp_1,zp_3,zp_5,\ldots,zp_{13}\}$ are $\{1,3,5,\ldots,13\}$, for $\{q_{14}q_{13},q_{13}q_{12},\ldots,q_3q_2\}$ are $\{4,6,8\ldots,26\}$, for $\{zp_{14},zp_{12},zp_{10},\ldots,zp_2\}$ are $\{47,49,51,\ldots,59\}$, for $\{zq_3,zq_5,\ldots zq_{13}\}$ are $\{19,21,\ldots,29\}$, for $\{zq_{14},zq_{12},\ldots,zq_2\}$ are $\{33,35,\ldots,45\}$, for $\{p_{14}p_{13},p_{13}p_{12},\ldots,p_2p_1\}$ are $\{34,36,38\ldots,58\}$ also for the edges $zq_{15},q_1q_2,p_{14}p_{15},q_1q_{16},q_{14}q_{15},q_{15}q_{16},zq_{16},zq_1,p_1p_{16},zp_{16},zq_{15},zp_{15}$ the induced edge labels are $2,15,17,28,30,31,32,60,61,62,63,64$ respectively. It can be observed that all the numbers from the set $\{1,2,3,\ldots,64\}$, where, $64=4l=m$, are covered for the edge labels, so, by the definition of Farey graceful labeling DW_{16} admits Farey graceful labeling.

In Figure 2, the graph with $l=15(odd)$, DW_{15} is labeled using the function χ with vertex labels as shown in the figure, similarly as in DW_{16}, the set of edge labels using the function χ^* in this graph is $\{1,2,3,\ldots,60\}$, where, $60=4l=m$, so, by the definition of Farey graceful labeling DW_{15} also admits Farey graceful labeling.

Analogously, using the given formula DW_l can be proved Farey graceful for any $l\ge 4$. Hence, the labeling χ is a Farey graceful labeling of DW_l.

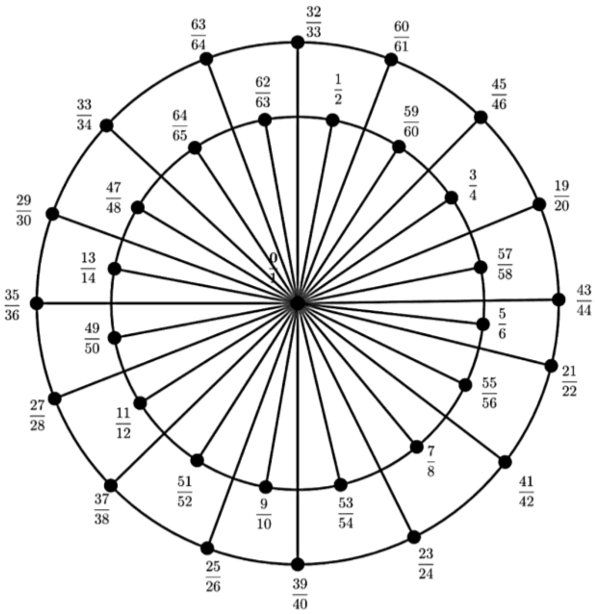

Fig 1: Farey Graceful labeling of DW_{16}

Theorem 2.2. All W^l graphs are Farey graceful for $l \in N$.

Proof. The graph W^l is constructed using two ladder graphs and a cycle graph. The cycle graph is obtained by joining the vertices of the two ladder graphs. The vertices on the left of the left ladder are represented by p_r, vertices on the right of the left ladder are represented by q_r, vertices on the left of the right ladder are represented by x_r and vertices on the right of the right ladder are represented by y_r. Additionally, a new vertex w is introduced and connected to the vertices $q_1, and\, x_1$. The graph W^l contains $4l + 5$ vertices and $6l+5=m$ edges. Define: $\chi: V(W^l) \to F_{m+1}$ by,

$$\chi(w)=\frac{0}{1}$$

$$\chi(p_1)=\frac{1}{2}$$

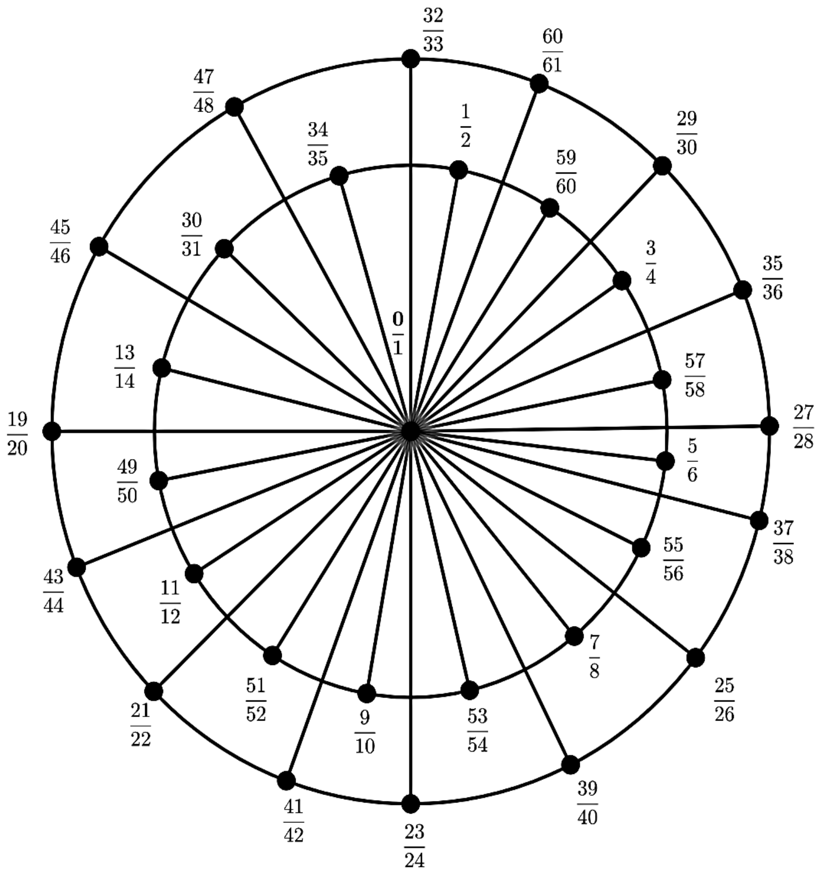

Fig 2: Farey Graceful labeling of DW_{15}

$$\chi(p_r)=\frac{3l+\frac{3}{2}+\frac{3}{2}r}{3l+\frac{5}{2}+\frac{3}{2}r},\ r \text{ be an odd number and} \begin{cases} 3\le r\le l,\ l \text{ is an odd number} \\ 3\le r\le l+1,\ l \text{ is even number} \end{cases}$$

$$\chi(p_r)=\frac{3l+5-\frac{3}{2}r}{3l+6-\frac{3}{2}r},\ r \text{ be even number and} \begin{cases} 2\le r\le l+1,\ l \text{ is an odd number} \\ 2\le r\le l,\ l \text{ is even number} \end{cases}$$

$$\chi(q_1)=\frac{6l+4}{6l+5}$$

$$\chi(q_r)=\frac{3l+\frac{11}{2}-\frac{3}{2}r}{3l+\frac{13}{2}-\frac{3}{2}r}, \text{ r be an odd number and} \begin{cases}3\le r\le l, \ l \text{ is an odd number}\\ 3\le r\le l+1, \ l \text{ is even number}\end{cases}$$

$$\chi(q_r)=\frac{3l+1+\frac{3}{2}r}{3l+2+\frac{3}{2}r}, \text{ r be even number and} \begin{cases}2\le r\le l+1, \ l \text{ is an odd number}\\ 2\le r\le l, \ l \text{ is even number}\end{cases}$$

$$\chi(x_r)=\frac{6l+\frac{13}{2}-\frac{3}{2}r}{6l+\frac{15}{2}-\frac{3}{2}r}, \text{ r be odd number and} \begin{cases}1\le r\le l, \ l \text{ is an odd number}\\ 1\le r\le l+1, \ l \text{ is even number}\end{cases}$$

$$\chi(x_r)=\frac{1+\frac{3}{2}r}{2+\frac{3}{2}r}, \text{ r be even number and} \begin{cases}2\le r\le l+1, \ l \text{ is an odd number}\\ 2\le r\le l, \ l \text{ is even number}\end{cases}$$

$$\chi(y_r)=\frac{\frac{3}{2}+\frac{3}{2}r}{\frac{5}{2}+\frac{3}{2}r}, \text{ r be odd number and} \begin{cases}1\le r\le l, \ l \text{ is an odd number}\\ 1\le r\le l+1, \ l \text{ is even number}\end{cases}$$

$$\chi(y_r)=\frac{6l+6-\frac{3}{2}r}{6l+7-\frac{3}{2}r}, \text{ r be even number and} \begin{cases}2\le r\le l+1, \ l \text{ is an odd number}\\ 2\le r\le l, \ l \text{ is even number}\end{cases}$$

The function χ of vertices induces the bijective function $\chi^* : E(W^l) \to \{1,2,3,\ldots,m\}$ of edges defined by $\chi^*\left(\frac{c_1}{d_1},\frac{c_2}{d_2}\right)=|c_1d_2-c_2d_1|$. In Figure 3, the graph with $l=7$, W^7 is labeled using the function χ with vertex labels as shown in the figure, the induced edge labels for the edge set $\{q_2q_3,q_3q_4,q_4q_5,\ldots,q_7q_8\}$ are $\{3,6,9,\ldots,18\}$, for $\{p_3q_3,p_4q_4,\ldots,p_8q_8\}$ are $\{5,8,11,\ldots,20\}$, for $\{p_3p_4,p_4p_5,\ldots,p_7p_8\}$ are $\{7,10,\ldots,19\}$, for $\{x_8y_8,x_7y_7,\ldots,x_1y_1\}$ are $\{23,26,29,\ldots,44\}$, for $\{y_8y_7,y_7y_6,\ldots,y_2y_1\}$ are $\{24,27,30,\ldots,42\}$, for $\{x_8x_7,x_7x_6,\ldots,x_2x_1\}$ are $\{25,28,31,\ldots,43\}$ also for the edges $q_1x_1,p_2p_3,p_2q_2,q_1q_2,p_1p_2,x_1y_1,p_1q_1,q_1w,x_1w$ the induced edge labels are $1,2,4,21,22,44,45,46,47$ respectively. It can be observed that all the numbers from the set $\{1,2,3,\ldots,47\}$, where, $47=6l+5=m$, are covered for the edge labels, so, by the definition of Farey graceful labeling W^7 admits Farey graceful labeling. Analogously, using the given formula W^l can be proved Farey graceful for all $l \in N$. Hence, the labeling χ is a Farey graceful labeling of W^l.

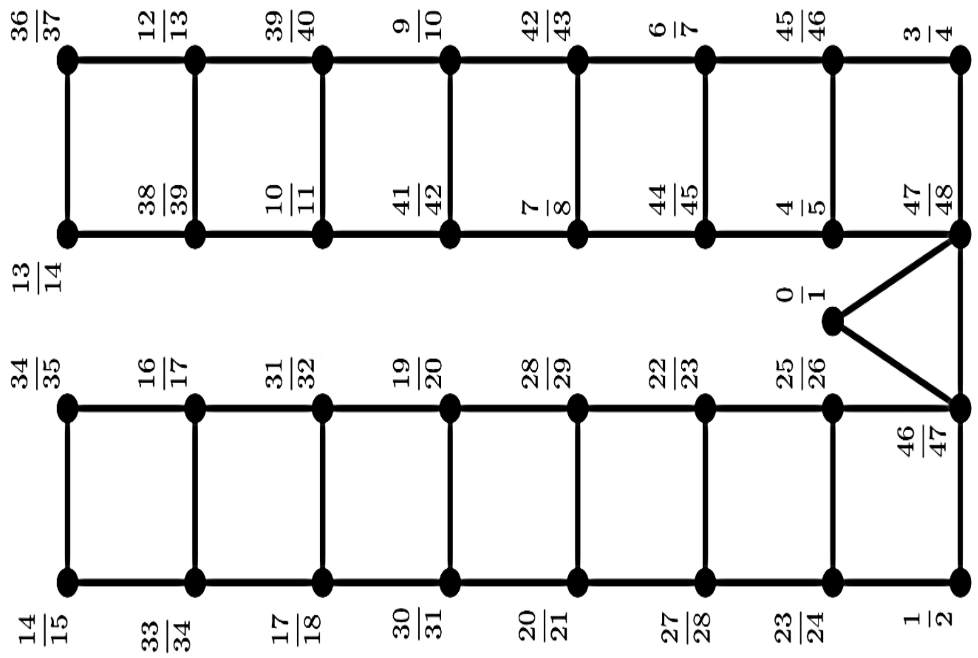

Fig 2: Farey graceful labeling of W^7

Theorem 2.3: When any arbitrary vertex in a cycle is switched, the new graph formed can be effectively assigned labels following the Farey graceful labeling.

Proof: The set of vertices of C_n be $\{v_0, v_1, v_2, \ldots, v_{n-1}\}$ and WLOG assume that the vertex v_0 is switched. The generated graph consists of number of edges $2n-5=m$ and the vertex labeling function $\chi: V \to F_{m+1}$ can be defined as:

Case 1: When n is even,

$$\chi(v_0)=\frac{0}{1}$$

$$\chi(v_1)=\frac{n-2}{n-1}$$

$$\chi(v_i)=\begin{cases}\dfrac{2n-i-3}{2n-i-2}, & i \text{ is even}\\[2mm] \dfrac{i-2}{i-1}, & i \text{ is odd}\end{cases}$$

Case 2: When n is odd,

$$\chi(v_0)=\frac{0}{1}$$

$$\chi(v_{n-1})=\frac{n-3}{n-2}$$

$$\chi(v_i)=\begin{cases}\dfrac{n-i-2}{n-i-1}, & i \text{ is even}\\[2mm] \dfrac{n+i-3}{n+i-2}, & i \text{ is odd}\end{cases}$$

The function χ of vertices induces the bijective function $\chi^* : E \to \{1,2,3,\ldots,m\}$ of edges defined by $\chi^*\left(\frac{c_1}{d_1}, \frac{c_2}{d_2}\right) = |c_1 d_2 - c_2 d_1|$. In Figure 4, the graph with $n=20(even)$, graph obtained by switching the vertex of C_{20} is labeled using the function χ with vertex labels as shown in the figure, the induced edge labels for the edge set $\{v_{19}v_{18}, v_{18}v_{17}, v_{17}v_{16}, \ldots, v_3v_2\}$ are $\{2,4,6,\ldots,34\}$, for $\{v_0v_3, v_0v_5, v_0v_7, \ldots, v_0v_{17}\}$ are $\{1,3,5,\ldots,15\}$, for $\{v_0v_{18}, v_0v_{16}, v_0v_{14}, \ldots, v_0v_2\}$ are $\{19,21,23,\ldots,35\}$, also for the edge v_1v_2 the induced edge label is 17. It can be observed that all the numbers from the set $\{1,2,3,\ldots,35\}$, where, $35=2n-5=m$, are covered for the edge labels, so, by the definition of Farey graceful labeling this graph admits Farey graceful labeling.

In Figure 5, the graph with $n=19(odd)$, graph obtained by switching the vertex of C_{19} is labeled using the function χ with vertex labels as shown in the figure, similarly as before, the set of edge labels using the function χ^* in this graph is $\{1,2,3,\ldots,33\}$, where, $33=2n-5=m$, so, by the definition of Farey graceful labeling this graph also admits Farey graceful labeling.

Analogously, using the given formula any graph obtained by switching the vertex of a cycle can be proved Farey graceful. Hence, the labeling χ is a Farey graceful labeling.

3. Applications of Farey graceful labeling

- **Classification of Graph Families**
  An important application of Farey graceful labeling is the classification of graphs. Establishing that a graph admits Farey graceful labeling allows it to be categorized within a broader class of labeled graphs. This approach is particularly useful for structured graphs such as ladder graphs, wheel graphs and double wheel graphs, where labeling techniques reveal their combinatorial behaviour.
- **Construction of New Farey Graceful Graphs**
  Farey graceful labeling supports the construction of new graphs from existing ones through operations such as vertex switching, edge subdivision and graph extension. In many cases, these operations preserve Farey graceful properties under suitable conditions. This contributes to the expansion of known Farey graceful graph families and strengthens the theoretical foundation of graph labeling.
- **Contribution to Graph Labeling Theory**
  Farey graceful labeling extends the classical concept of graceful labeling by incorporating ideas from number theory, particularly Farey sequences. This interaction opens new research directions in generalized and fractional labeling schemes and contributes to the development of advanced labeling techniques in graph theory.
- **Educational and Mathematical Significance**
  From an academic perspective, Farey graceful labeling serves as a valuable example of the interplay between number theory and graph theory. It is useful for teaching advanced topics in discrete mathematics and motivates further research on rational labelings of graphs.

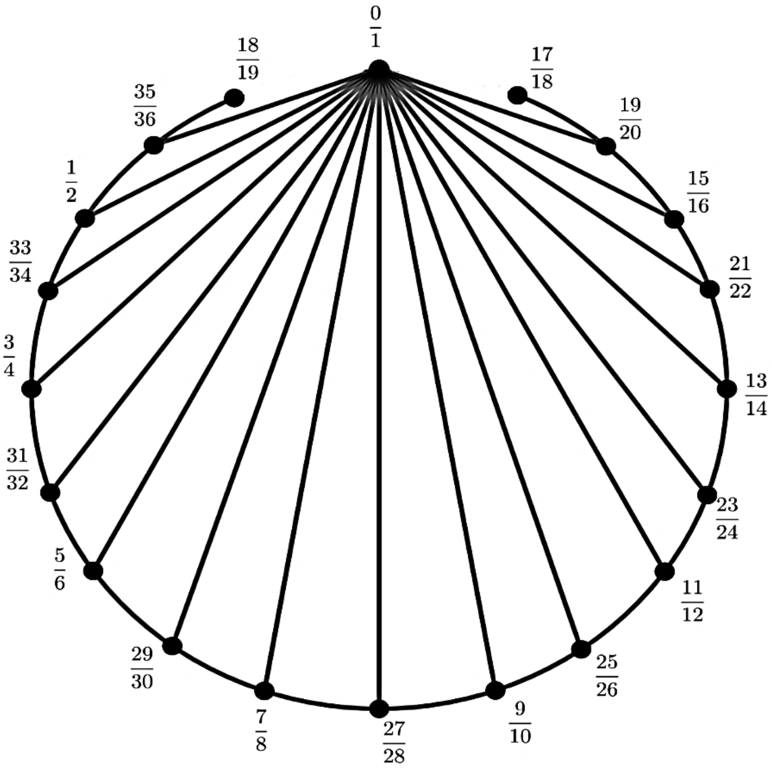

Fig 4: Farey graceful labeling of graph obtained by vertex switching of C_{20}

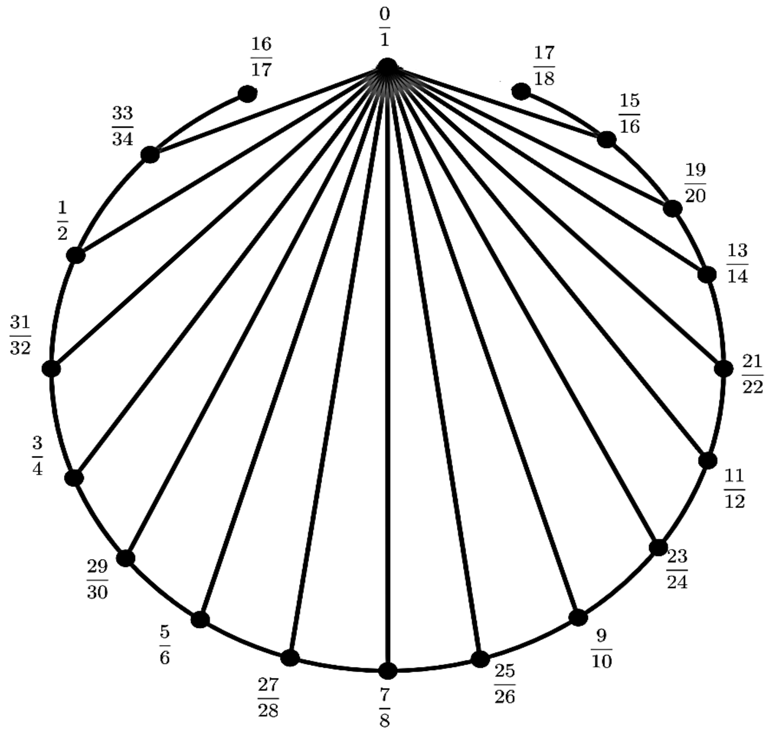

Fig 5: Farey graceful labeling of graph obtained by vertex switching of C_{19}

4. Future Research Directions of Farey Graceful Labeling

Farey graceful labeling is a relatively recent and evolving area in graph labeling theory. Although several classes of graphs have been shown to admit Farey graceful labeling, many theoretical and applied aspects remain unexplored. The following directions suggest promising avenues for future research.

a. Characterization of New Graph Families

One important direction is to identify and characterize broader classes of graphs that admit Farey graceful labeling. These include, Product graphs, Graph powers, Subdivision and expansion graphs, Vertex-switched and edge-switched graphs. Establishing necessary and sufficient conditions for Farey graceful labeling remains an open and challenging problem.

b. Farey Graceful Labeling under Graph Operations

Further work can focus on the behaviour of Farey graceful labeling under standard graph operations such as, Vertex switching and edge switching, Graph union, join and corona, Line graphs and total graphs. Studying the preservation or transformation of Farey graceful properties under these operations may lead to general labeling theorems.

c. Algorithmic and Computational Aspects

The development of efficient algorithms to determine whether a given graph is Farey graceful is an important research direction. Future studies may address Polynomial-time algorithms for specific graph classes, Computational complexity of Farey graceful labeling, Automated generation of Farey graceful labelings for large graphs. These studies would enhance practical usability and computational understanding.

d. Extensions to Directed and Weighted Graphs

Another promising area is extending Farey graceful labeling to, directed graphs, weighted graphs, Signed graphs, Fuzzy graphs. Such extensions could significantly broaden the applicability of Farey graceful labeling in real-world modeling.

e. Applications in Network Science and Engineering

Future research may explore practical applications of Farey graceful labeling in, Communication and sensor networks, Frequency assignment problems, Secure data transmission, Load balancing and optimization problems. These applications could bridge the gap between theoretical graph labeling and applied network design.

5. Conclusion

This paper presents new results on Farey graceful labeling for specific classes of graphs. We have shown that the W-graph admits a Farey graceful labeling through a systematic construction that ensures the uniqueness of induced edge labels. This result extends the applicability of Farey graceful labeling to a previously unexplored graph structure. Additionally, we proved that the double wheel graph and graph obtained by vertex switching of cycle are Farey graceful by assigning appropriate Farey fractions to their vertices and verifying that the induced edge labels satisfy the defining conditions. This demonstrates that Farey graceful labeling can be achieved even in graphs with intricate cyclic and radial configurations.

The results obtained in this study broaden the class of graphs known to admit Farey graceful labeling and contribute to the theoretical development of graph labeling. These findings open avenues for future research on Farey graceful labeling under other graph operations and for more complex graph families.

Acknowledgments

The authors thank the institute, Hemwati Nandan Bahuguna Garhwal University, for their support.

References

Acharya BD (1983) Construction of graceful graphs. In: Acharya BD and Rao SR (Eds.) *Graph Theory and Applications*. Springer, Berlin. pp.1–12.

Bloom GS and Golomb SW (1977) Numbered complete graphs, unusual rulers, and assorted applications. In: Alspach B and Wallis WD (Eds.) *Theory and Applications of Graphs*. Springer, Berlin. pp.53–65.

Burton D (2010) Elementary Number Theory. McGraw Hill

Ellingham MN (1991) Vertex-switching, isomorphism and pseudosimilarity. Journal of Graph Theory. 15(6). 563-572.

Gallian JA (2023) A dynamic survey of graph labeling Electronic. J. Combinatorics.

Harary F (1988) *Graph theory*. Addison-Wesley Publishing Company, Reading, Massachusetts, USA. 274 Pages.

Kumar A, Mishra D, Srivastava VK (2024) On Farey Graceful Labeling. National Academy Science Letters. 47. 303–308.

Kumar A, Kumar A and Tyagi S (2025) Labeling of graphs using rational numbers. Mathematical Forum. 33(1).

Kumar A, Kumar A, Tyagi S, Gupta N (2024) Farey graceful labeling of graphs and its applications. National Academy Science Letters.

Kumar A, Tyagi S and Kumar A (2024) Labeling of graphs using taxi-cab metric. Journal of Mountain Research. 19(2). 259-263.

Kumar A, Gupta N, Kumar A, Tyagi S, Kumar V (2024) On Farey Edge Graceful Labeling. IAENG International Journal of Applied Mathematics. 54(11). 2484-2490.

Kumar A, Gupta N, Kumar A, Tyagi S, Kumar V, Gupta T (2024) Farey Edge Graceful Labeling on Cycle, Star and Corona Product of Graph. IAENG International Journal of Applied Mathematics. 55(8). 2581-2587.

Rosa A (1967) On certain valuations of the vertices of a graph. Theory of Graphs (Internet Symposium, Rome, July 1966). Gordon and Breach. N. Y. and Dunod Paris. 349-355.

Seoud MA and Salim MA (2007) Switching equivalence and graph labelings. *J. Combin. Math. Combin. Comput.* 61: 95–104.

Yilmaz S and Cahit I (2010) On cordial and related labelings of wheel graphs. *Ars Combinatoria*. 97: 227–234.

Zhang Z and Liu J (2015) Graceful and near graceful labelings of wheel-related graphs. *Discrete Mathematics*. 338: 1329–1336.