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CLUSTER SHELL MODEL WAVE FUNCTION: STRUCTURE OF ^{6}LI NUCLEUS

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Abstract: In this paper cluster model wave function for ^{6}Li using Shell Model with definite parity and angular momentum is written along with cluster co-ordinates, which are relative to the center-of-mass of various clusters and involve with parameters. These parameters can be adjusted to some extent to obtain predictions close to experimental properties. The cluster model wave function is written along with resonating group method (RGM) and the Complex Generator Coordinate Technique (CGCT). The Complex Generator Coordinate Technique allows the transformation of the cluster model wave function written in terms of cluster co-ordinates into anti-symmetrized product of single particle wave function. This wave function is written in terms of single particle co-ordinates, the center-of-mass co-ordinates, parameter coordinates and generator coordinates.

Key words: Cluster model, ^{6}Li nucleus, RGM, CGCT, Wave function

Introduction

^{6}Li is considered as one of the stable light nuclei with a spin 1 in the ground state. ^{6}Li nucleus consists of 3 protons and 3 neutrons. It is considered as a two-body problem (Griffin and Wheeler, 1957) consisting of $\alpha + d$ (alpha cluster and deuteron cluster). The wave function is now written employing Resonating group method (Horiuchi, 1970) along with Complex generator coordinate technique (Hill and Wheeler, 1953). It may be assumed to a good approximation that the cluster remains unexcited and undistorted like a free alpha particle in ^{6}Li nucleus. The spin and parity of the nucleus in ground state is $J = 1^+$. The nuclear shell model starts from an average potential with a shape something between square well and harmonic oscillator (Tang, 1978). We have used here a more realistic but at the same time a complicated potential known as Wood-Saxon potential (Woods, 1954).

Cluster model Wave function using Shell model

The cluster model (Margenau, 1941) wave function for ^{6}Li nucleus in ground state is written as

$$\Phi_{1M} = A\left[\Phi(\alpha)\Phi(d)\Sigma_{ml+ms}\; C\left(1.m_l,\frac{1}{2},m_s,1,M\right)x_{l_{m_l}}(\overline{R})\xi_{\frac{1}{2}}m_s\right] \quad (1)$$

A is antisymmetrization operator which consists of inter cluster (α cluster) and intra-cluster (α and deuteron cluster) exchange terms.

$\Phi(\alpha)$ is a wave function of α cluster.

The wave function of each cluster is described by a translational of invariant of Harmonic Oscillator Shell model function

$$\Phi(\alpha) = \overline{\Phi}_s(\alpha)\,\xi_\alpha\,(\sigma,\tau) \quad (2)$$

Where, $\bar{\Phi}_s$ (α) represents the spatial part of α-cluster

ξ_α (σ, τ) Represents spin isospin part of α-cluster

Φ_s (d)represents spatial part of deuteron

$x_{l_{m_l}}(\bar{R})$ is a relative motion wave function between alpha and deuteron cluster.

Here antisymmetrization operator consists of inter- and intra-cluster exchange terms (Perring 1956). The internal structure of each cluster is described by a translationally invariant Harmonic Oscillator Shell model function of the lowest configuration.

In order to convert this wave function into anti-symmetrized product of single particle wave function we are introducing an integral representation for Φ.

$$\Phi_{1M} = \int A\,[\Phi_s(\alpha,R'_\alpha)\,\Phi_s(d,R'_d)\,\delta(R_\alpha - R'_\alpha)\,\delta(r_d - r'_d)\xi_\alpha\xi_d x_{l_{m_l}}(R'_\alpha - r'_d)_{cm}\left(\frac{4R'_\alpha + 2r_d}{6}\right)]\,dR'_\alpha\,dr'_d$$

(3)

The terms having double prime denote parameter co-ordinates.

Using integral representation for the delta function

$$\delta(R_K - R'_K) = \left(\frac{1}{2\pi}\right)^3 \int \exp[i\,S''_K.(R_K - R'_K)]\,dS''_K$$

(4)

where K = alpha and deuteron cluster

S_K'' denotes the generator co-ordinates

$$\Phi_j(r_j - R'_\alpha) = exp.\left[\frac{-\alpha}{2}(r_j - R'_\alpha)^2\right]$$

$$\Phi_d(r_i - R'_d) = exp.\left[\frac{-\beta}{2}(r_i - R'_d)^2\right]$$

(5)

Where j = 1 to 4 for alpha cluster and r_i = 5, 6 for deuteron cluster.

$$\Phi_{1M} = \int A\,\Pi_{j=1}^{4} exp\left[\frac{-\alpha}{2}(r_j - R'_\alpha)^2\right]\Pi_{i=5}^{6} exp.\left[\frac{-\beta}{2}(r_i - R'_d)^2\right]$$

$$exp.\left[i\,S''_a(R_a - R'_\alpha)\right] exp.\left[i\,S''_d(R_d - R'_d)\right]\xi_\alpha\xi_d x_{l_{m_l}}(R'_\alpha - R'_d)Z_{cm}\left(\frac{4R'_\alpha + 2R'_d}{6}\right)\,dR'_\alpha\,dS''_\alpha\,dR'_\alpha S''_\alpha \quad (6)$$

In order to solve this integral we will make some transformations in above equation as

$$\frac{1}{4\alpha}\bar{s}_a'' - i\,R'_\alpha = \bar{P}$$

(7)

$$\frac{1}{2\beta}\bar{s}_d'' - i\,R'_d = \bar{Q}$$

Using Jacobian of transformation |J|, one can write

$$\bar{d}S''_\alpha \bar{d}S''_d \bar{d}R'_\alpha \bar{d}R'_d = |\,J\,|\,d\bar{P}\bar{d}Q\bar{d}R''_{cm}d\bar{R}''_1 \quad (8)$$

|J| can be calculated by the use of determinant.

Due to the translational invariant Hamiltonian one can choose the center-of-mass wave function arbitrarily one can choose $Z(\bar{R}''_{cm})$ in the form

$$Z(\bar{R}''_{cm}) = \exp.\left[-R''^2_{cm}\left(\frac{4\alpha + 2\beta}{2}\right)\right] \quad (9)$$

Making use of equations (8) and (9) we can solve and normalize equation (6).

Conclusion

In this way using Cluster Model wave function along with CGCT, concept of antisymmetrization and in center of mass system, we have constructed a wave function for ^{6}Li nucleus. The determined wave function can be used to calculate theoretically several important microscopic parameters like charge form factor, root mean square (r.m.s.) radius and quadrupole moment of the nucleus. Such wave functions for the ground state of ^{9}Be nucleus (Sinha, 2011) and ^{5}He nucleus (Sinha, 2013) are formulated earlier as well. The charge form factor for ^{5}He nucleus has further been formulated (Sinha, 2019). Similarly, our future objective will be to determine the parameters for ^{6}Li nucleus.

The studies of ^{6}Li nucleus has been of interest as it is an important and valuable isotope in nuclear physics because when it is bombarded with neutrons, tritium is produced and absorbs neutrons in nuclear fusion reactions.

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