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A CHARGE FORM FACTOR (CFF) OF ^{5}He NUCLEUS

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Abstract: High energy electron scattering is a very powerful tool for studying geometrical details of nuclear structure. The studies provide information on static distribution of charge and magnetization in nuclei. As the interaction is relatively weak so that in the scattering process the internal structure of the target nucleus is not significantly disturbed. Using electrons as projectile, we can study how transition matrix elements vary with q^2 and map out the Fourier transforms of the transition charge and current densities called Form Factors or Structure factors. In the high energy electron scattering we can know the details of the spatial distribution of transition charge and current density. In this paper we have formulated CFF for ^{5}He nucleus

Keywords: Charge Form Factor (CFF), Resonating group method (RGM).

Introduction

In electron scattering as the momentum transfer q is variable. The static moments are multiplied by form factors which are functions of q when we calculate the elastic scattering cross section in the Born approximation. Since the Born approximation is used, the resulting expressions are valid for the light nuclei up to the s-d shells only. The static or transition multipole moments of densities which appear in the theoretical expressions of the form factors can be calculated from a nuclear shell model.

Formulation and calculation of charge form factor

The antisymmetrized wave function of ground state of ^{5}He nucleus can be written with definite spin and parity using the cluster model along with the Resonating Group Method (RGM) and Generator Coordinate Method (GCM). ^{5}He nucleus in ground state is considered to consist of one alpha cluster and a neutron cluster. complex generator coordinate technique (CGCT) is applied to the cluster model wave function to transform it into antisymmetrized product of single particle wave function.

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The wave function of ^{5}He in the ground state can be written as:

$$\Phi_{\frac{3}{2}\frac{3}{2}} = \left(\frac{\alpha^5}{\pi^4}\right) \int A \left[\prod_{i=1}^{4} \xi_a \, \xi_n \exp\left[-\frac{\alpha}{2}(\bar{r}_i - i\bar{P})^2 \right] \exp\left[-\frac{\alpha}{2}(\bar{r}_5 + 4i\bar{P})^2\right] \right.$$

$$\left. \chi_{11}\,(\bar{R}')\, \exp\left[-2\alpha \left(\sqrt{5}\,\bar{P} + \frac{1}{\sqrt{5}}\bar{R}'\right)^2\right] \right] d\bar{P}\,.d\bar{R}' \qquad \text{-- (1.0)}$$

Charge form factor may be represented as:

$$|\,F_{ch}\,(q^2)\,|^2 = \frac{1}{Z^2}\frac{1}{(2J_i+1)} \sum_{M_i\,M_J} |\, \langle J_f M_f \,|\, \sum_J \frac{1+\tau_{Z\,(j)}}{2} \exp(i\,\bar{q}\;\bar{r}_j)|\, J_i M_i \rangle\,|^2 \qquad \text{-- (2.0)}$$

Where $|F_{Ch}\,(q^2)|^2$ is the square of charge form factor of the nucleus.

In CGCT the correction for the center-of-mass motion is made exactly. When recoil effects are considered one still uses the same charge density $\hat{\rho}\,(r)$ as before but with $\bar{r}$ considered the variable in the rest, system of the nucleus i.e., $\bar{r}$ is measured from the center-of-mass of the nucleus (in laboratory system) i.e.

$$\bar{r} = \bar{r}_{lab} - \bar{R}_{cm}$$

or $\bar{r}' = \bar{r} - R_{cm}$

eq. (2.0) may be written as:

$$|F_{ch}\,(q^2)|^2 = \frac{1}{Z^2}\frac{1}{(2J_i+1)} \sum_{M_i\,M_f} |\, \langle J_f M_f \,|\, \sum_J \frac{1+\tau_{Z\,(j)}}{2} \exp(-i\,q(\bar{r}_j - \bar{R}_{cm})|\, J_i M_i \rangle\,|^2 \qquad \text{-- (3.0)}$$

Where $J_i = J_f$

For the ground state of ^{5}He nucleus the angular momentum J_i = 3/2 so that above equation becomes

$$|F_c\,(q^2)|^2 - \frac{1}{4Z^2} \sum_{M_i\,M_i} |\, \langle \Phi_{\frac{3}{2}M'_i} \,|\, \sum_{J=2.4} \exp\{-i\bar{q}(\bar{r}_j - \bar{R}_{cm}\}|\, \Phi_{\frac{3}{2}M'_i} \rangle\,|^2 \qquad \text{-- (4.0)}$$

Where $\Phi_{3/2.}M_i$ or $\Phi_{3/2.}M_i$ are the translationally invariant normalized wave functions for the ground state of ^{5}He nucleus.

For un normalized wave functions the matrix element *M* in eq. (4.0) may be written as

$$M = \frac{\langle \Phi_{\frac{3}{2}M'_i} \,|\, \sum_{J=2.4} \exp\{-i\bar{q}(\bar{r}_j - \bar{R}_{cm}\}|\, \Phi_{\frac{3}{2}M'_i} \rangle_R}{<\Phi_{\frac{3}{2}M'_i} \,|\, \Phi_{\frac{3}{2}M_i} >_R} \qquad \text{-- (5.0)}$$

Where the suffix R implies integration over all the internal coordinates of the nucleus in ^{5}He. Multiplying and dividing the right-hand-side of eq. (5.0) by the wave function of the center-of-mass of ^{5}He nucleus. Eq (5.0) can be written as:

$$M = \frac{\langle \Phi_{\frac{3}{2}M'_i} \,|\, \sum_{J=2.4} \exp\{-i\bar{q}.\,\bar{r}_j\}|\, \Phi_{\frac{3}{2}M'_i} \rangle}{\langle \Phi_{\frac{3}{2}}M'_i |\, \exp\{-i\bar{q}.\,\bar{r}_j\}|\, \Phi_{\frac{3}{2}}M_i \rangle} \qquad \text{-- (6.0)}$$

Where $\Phi_{\frac{3}{2}M_i}$ is the wave function, of the ground state of ^{5}He nucleus[1] which includes the wave function for the motion of the center-of-mass of ^{5}He.

The possible values of M_i and M_i' in eqs. (4.0) to (6.0) are 3/2, 1/2 – 1/2 and -3/2. In carrying out the summation over M_i and M_i' in eq. (4.0) with the matrix element M given by eq. (6.0).

In order to take into account the finite size of the protons in ^{5}He nucleus the expression (2.0) for the square of the charge form factor is multiplied by the square of the charge form factor of the proton given by [3],[4]

$$F_{ch}\,(p) = 0.5 \left[\frac{2.5}{1+\frac{q^2}{15.7}} - \frac{1.6}{1+\frac{q^2}{26.7}} + \frac{1.16}{1+\frac{q^2}{8.19}} \right] - 0.03 \qquad \text{-- (7.0)}$$

The square of the charge form factor $|F_{ch}\,(q^2\ {}^5He)|^2$ for the ground state of ^{5}He nucleus is thus

$$|F_{ch}\,(q^2\,,\ {}^5He)|^2 - |F_c\,(q^2)|^2\ |F_{ch}\,(p)|^2 \qquad \text{-- (8.0)}$$

Conclusion

Here we have formulated CFF for ^{5}He nucleus. We have considered all the possible four angular momentum values in this formulation. Since the ground state of ^{5}He nucleus is unbound, no experimental data for the CFF exists for this nucleus. Hence, for purpose of comparison the experimental data of Frosch et.al [5] for $|F_{ch}(q^2)|^2$ available for the nearby nucleus i.e., 4He in the range of q^2 from 0 to 18fm^{-2}. In this investigation the main emphasis was to develop a method for the calculation of the CFF for ^{5}He nuclei using cluster model wave function along with the complex coordinate technique. The antisymmetrization of the wave function and motion of the center-of-mass have been properly taken into account.

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