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ON SOME TYPES OF AFFINE MOTIONS IN TACHIBANA RECURRENT SPACE OF SECOND ORDER

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ABSTRACT

In the present paper, we have defined and studied some types of affine motions in Tachibana recurrent space of second order and several theorems have been derived.

KEY WORDS : Almost Tachibana Space, Tachibana Space, Reimannian matric, Tachibana recurrent space of second order, Lie-derivative, Affine motions.

INTRODUCTION

Let us consider $m\,(=2n)$ dimentional Real mainfold M_{2n} of differentiability class (C^r) with respect to an allowable Co-ordinate system :

$$\left(x^i, x^{\bar{i}}\right) \circ \left(x^1, x^2, \ldots\ldots, x^{n+1}, x^{\bar{1}}, x^{\bar{2}}, \ldots\ldots, x^{\overline{n+1}}\right)$$

If there exist a mixed tensor $F_j^h\left(x^i, x^{\bar{i}}\right)$ of class C^r, which satisfies

$$F_j^i F_i^h = -A_j^h. \qquad \ldots(1.1)$$

and the Riemannian matric g_{ij} satisfying :

$$dS^2 = g_{ij}(x, \bar{x})\, dx^i\, dx^{\bar{j}}. \qquad \ldots(1.2)$$

which also satisfies the condition.

$$F_{ih,j} + F_{jh,i} = 0. \qquad \ldots(1.3)$$

then, the space is called an almost Tachibana Space. If the conditions

$$\frac{\partial^2 x^h}{\partial x^j \partial x^i} + \frac{\partial x^k}{\partial x^i} g^{hs} \partial_k g_{js} - \frac{\partial x^h}{\partial x^k} g^{ks} \partial_i g_{js} = 0 \qquad \ldots(1.4)$$

and
$$\frac{\partial^2 x^{\bar{h}}}{\partial x^{\bar{j}} \partial x^{\bar{i}}} + \frac{\partial x^{\bar{k}}}{\partial x^{\bar{i}}} g^{\bar{h}s} \partial_{\bar{k}} g_{\bar{j}s} - \frac{\partial x^{\bar{h}}}{\partial x^{\bar{k}}} g^{\bar{k}s} \partial_{\bar{i}} g_{\bar{j}s} = 0 \qquad \ldots(1.5)$$

are satisfied, then the space is said to be a Tachibana space.

DEFINITION

A space T_n is said to be a Tachibana recurrent space of second order, if the following condition is satisfied [2]

$$R^h_{ijk.mn} = \lambda_{mn} R^h_{ijk}. \qquad \ldots(1.6)$$

where λ_{mn} is non-zero and in general, non-symmetric covariant tensor of order 2. It is denoted by ${}^2T_n$-space.

Introducing affine motions

$$\bar{x}^i = x^i + \upsilon^i(x)\,\delta t. \qquad \ldots(1.7)$$

of special types, we have studied the essential properties of the space [3]. As a continuation of our study in this paper, we shall try to investigate on the space admitting an affine motion (1.7) of ${}^2T_n$-spaces, characterized by

$$\upsilon_{.mn}{}^i = K_{mn}\,\upsilon^i. \qquad \ldots(1.8)$$

where in general, we assume that $K_{mn} \neq \lambda_{mn}$. Being (1.7) an affine motion, it is characterized by

$$\pounds_\upsilon \Gamma^h_{ij} = \upsilon^h_{.ij} + R^h_{ijk}\,\upsilon^k = 0. \qquad \ldots(1.9)$$

Here, $\pounds_\upsilon$ denotes the so called Lie-derivative with regard to (1.9).

AFFINE MOTION IN TACHIBANA RECURRENT SPACE OF SECOND ORDER

If the space ${}^2T_n$ admits affine motion, the condition (1.8) must be integrable and as its integrability condition, we have

$$\pounds_\upsilon R^h_{ijk} = 0. \qquad \ldots(2.1)$$

According to a method by E. Cartan, taking $\upsilon_{.i}{}^h = R^h_{ijk}\,F^{jk}$, for a non-symmetric tensor F^{jk}, condition (2.1) may be replaced by

$$V^a R^h_{ijk.a} = C R^h_{ijk}. \qquad \ldots(2.2)$$

where $C = A_{mn} F^{mn}$ and $A_{mn} = \lambda_{mn} - \lambda_{nm}$.

Occurrence of Two Cases: Under the existence of affine motion (1.7), from (1.8) and (1.9), we have

$$K_{ij}\upsilon^h + R^h_{ijk}\,\upsilon^k = 0. \qquad \ldots(2.3)$$

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