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A FIXED POINT THEOREM FOR A FINITE PRODUCT OF METRIC SPACES

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ABSTRACT

In this paper we establish a fixed point theorem for a pair of maps on a finite product of metric spaces. Our result extend and generalize the results of Prešić 1965, and Ćirić-Prešić 2007.

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1. INTRODUCTION

Amongs the various generalization of well known Banach's contraction principle, in 1965 Prešić 1965 gave a contractive condition on metric spaces and proved the following result.

THEOREM 1.1 Prešić 1965. Let (X,d) be a complete metric space, k a positive integer and $S: X^k \to X$ a mapping satisfying the following contractive type condition

(1.1) $d(S(x_1,x_2,\ldots,x_k),S(x_2,x_3,\ldots x_k,x_{k+1})) \le q_1 d(x_1,x_2)+q_2 d(x_2,x_3)+\ldots+q_k d(x_k,x_{x+1})$,

for every $x_1,\ldots,x_{k+1}$ in X, where $q_1,q_2,\ldots,q_k$ are non-negative constants such that $q_1+q_2+\ldots+q_k<1$. Then there exists a unique point x in X such that $S(x,\ldots,x)=x$ Moreover if $x_1,x_2,\ldots,x_k$ are arbitrary points in and for, $n \in N$

$$x_{n+k} = S(x_n,x_{n+1},\ldots,x_{n+k-1}) ,$$

then the sequence is convergent and

Further Ćirić – Prešić 2007 generalize the Theorem 1.1 stated as follows:

THEOREM 1.2 Ćirić – Prešić 2007 Let (X,d) be a complete metric space, k a positive integer and $S:X^k \to X$ a mapping satisfying the following contractive type condition

(1.2) $d(S(x_1,x_2,...,x_k),S(x_2,x_3,...,x_k,x_{k+1})) \le \lambda \max\{d(x_i,x_{i+1}):1\le i\le k\}$,

for every $x_1,...,x_{k+1}$ in X and $\lambda \in [0,1)$ is constant. Then there exists a point x in X such that . Moreover if are arbitrary points in and for $n\in N$,

$$x_{n+k} = S(x_n,x_{n+1},...,x_{n+k-1}),$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim x_n = S(\lim x_n, \lim x_n,...,\lim x_n).$$

If in addition we suppose that on diagonal $\Delta \subset X^k$,

$$d(S(u,...,u),S(v,...,v)) < d(u,v)$$

holds for all $u,v\in X$, with $u \ne v$, then x is the unique point in X with

$$S(x,...,x) = x.$$

In this paper we will extend the contractive condition (1.2) for two maps by using the proof technique of Ćirić-Prešić 2007. Our result generalize the results of Prešić 1965 and Ćirić-Prešić 2007.

2. MAIN RESULT

Now we state our main theorem.

THEOREM 2.1. Let (X,d) be a complete metric space, k a positive integer and $S,T:X^k \to X$ be mappings satisfying the following condition

(2.1) $d(S(x_1,x_2,...,x_k),T(x_2,x_3,...,x_k,x_{k+1})) \le \lambda \max\{d(x_i,x_{i+1}):1\le i\le k\}$,

for every $x_1,...,x_{k+1}$ in X and $\lambda \in [0,1)$ is constant. Then there exists a point x in such that

$$T(x,...,x) = x = S(x,...,x).$$

Moreover if $x_1,x_2,...,x_k$ are arbitrary points in X and for $n\in N$,

$$x_{2n+k+1} = T(x_{2n+1},x_{2n+2},...,x_{2n+k}),$$

$$x_{2n+k+2} = S(x_{2n+2},x_{2n+3},...,x_{2n+k+1}),$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$S(x,y)=\frac{x}{3}+2$, if $(x,y)\in[2,3]\times[2,3]$,

$S(x,y)=\frac{x+y}{6}-\frac{1}{3}$, if $(x,y)\in[0,1]\times[2,3]$ or $(x,y)\in[2,3]\times[0,1]$

and

$T(x,y)=\frac{y}{3}$, if $(x,y)\in[0,1]\times[0,1]$,

$T(x,y)=\frac{y}{3}+2$, if $(x,y)\in[2,3]\times[2,3]$,

$T(x,y)=\frac{x+y}{6}-\frac{1}{3}$, if $(x,y)\in[0,1]\times[2,3]$ or $(x,y)\in[2,3]\times[0,1]$.

Then for any $x,y\in[0,1]$ we have $S(x,y)=z\in[0,1]$ or $T(x,y)=z\in[0,1]$ and for $x,y\in[2,3]$ we have $S(x,y)=z\in[2,3]$ or $T(x,y)=z\in[2,3]$. Thus for or , we have

$$d(S(x,y),T(y,z))=\left|\frac{x}{3}-\frac{z}{3}\right|$$

$$\leq\tfrac{2}{3}\max\{d(x,y),d(y,z)\}.$$

For $(x,y)\in[0,1]\times[2,3]$ or $(x,y)\in[2,3]\times[0,1]$ we have $S(x,y)=z\in[0,1]$ or $T(x,y)=z\in[0,1]$.

Therefore, if $x\in[0,1]$ and $y\in[2,3]$, then

$$d(S(x,y),T(y,z))=\left|\frac{x+y}{6}-\frac{1}{3}-\frac{y+z}{6}+\frac{1}{3}\right|$$

$$\leq\left|\frac{x-y}{6}+\frac{y-z}{6}\right|$$

$$\leq\tfrac{2}{3}\max\{d(x,y),d(y,z)\}.$$

If $x\in[2,3]$ and $y\in[0,1]$ then

$$d(S(x,y),T(y,z))=\left|\frac{x+y}{6}-\frac{1}{3}-\frac{z}{3}\right|$$

$$\leq \left|\frac{x+y-2z}{6}-\frac{1}{3}\right|$$

$$< \left|\frac{x-z+y-z}{6}\right|$$

$$\leq \tfrac{2}{3}\max\{d(x,y),d(y,z)\}.$$

Thus T satisfies (2.1) with $\lambda = 2/3$, but for $x = 0$ and $y = 3$ we have $T(0,0) = 0 = S(0,0)$ and $T(3,3) = 3 = S(3,3)$.

EXAMPLE 2.2. Let $X = [0,1]$ with usual metric d and let $S,T : X^2 \to X$ be mappings defined by

$$S(x,y) = \frac{x}{4} \quad , \quad T(x,y) = \frac{y}{4},$$

satisfying all the condition of Theorem 2.1 with $\lambda = 1/2$ and there is a unique point 0 such that $S(0,0) = 0 = T(0,0)$.

REMARK 2.1. We get the Theorem 1.2 by setting $S = T$ in Theorem 2.1.

REMARK 2.2. In view of the Remark 1 of Ćirić - Prešić 2007, the condition (1.1) implies the conditions (2.1) and (2.3) with S= T.

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