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ON DECOMPOSITION OF CURVATURE TENSOR FIELDS IN A TACHIBANA RECURRENT SPACE

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ABSTRACT

Takano (1967) has defined and studied the decomposition of curvature tensor field in a recurrent space and several theorems have been derived. Further, Sinha and Singh (1970) have studied the decomposition of recurrent curvature tensor fields in a Finsler space and serval interesting results have been obtained. The present paper is devoted to the study of decomposition of curvature tensor fields in a Tachibana recurrent space in which we have studied the relation between the recurrence vector and other tensors into which the recurrent curvature tensor field has been decomposed.

INTRODUCTION

Here, we shall firstly define Kaehler and Tachibana spaces and give some preliminary formulae, which are pre-requisites to understand such spaces.

An n(=2m) dimensional Kaehlerian space k_n is a Riemannian spacem, which admits a structure tensor field F^h_i, satisfying the relations

$$F^h_j F^i_h = -\delta^{i*)}_j , \qquad \text{.... (1.1)}$$

$$F_{ij} = -F_{ji} , (F_{ij} \overset{def}{=} F^a_i g_{aj}) \qquad \text{.... (1.2)}$$

and

$$\nabla_j F^h_i = 0, \qquad \text{.... (1.3)}$$

where the '∇' denotés the operator of covariant differentiation with respect to the metric tensor g_{ij} of the Riemannian space.

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*) *All the Latin indices run over the same range from 1 to n.*

**) $\partial_i = \partial/\partial x^i$ *, where* $\{x^i\}$ *denotes the real local co-ordinates.*

$$C^h_{ijk} = \frac{1}{n+2} [R_{ik}\, \delta^h_j - R_{jk}\, \delta^h_i + S_{ik}\, F^h_j - S_{jk}\, F^h_i + 2S_{ij}\, F^h_k]. \quad \text{... (2.22)}$$

Contracting the indices h and k in (2.1), we have

$$R_{ij} = v^k\, x_i\, F_{jk}. \quad \text{.... (2.23)}$$

In view of equation (2.23), we obtain

$$S_{ij} = F^l_i\, v^n\, x_l\, F_{jn}\,. \quad \text{.... (2.24)}$$

Making use of relations (2.23) and (2.24) in equation (2.22), we have

$$C^h_{ijk} = \frac{1}{n+1} [F_{kn}\, v^n\, \{(x_i\, \delta^h_j - x_j\, \delta^h_i) + x_l\, (F^h_j\, F^l_i - F_i^h\, F^l_j)\} + 2v^n\, x_l\, F_{jn}\, F^l_i\, F^h_k]. \quad \text{.... (2.25)}$$

From (2.22), it is clear that $P^h_{ijk} = R^h_{ijk}$, If $C^h_{ijk} = 0$, which in view of equation (2.25) becomes

$$F_{kn}\, v^n\, \{(x_i\, \delta^h_j - x_j\, \delta^h_i) + x_l\, (F^h_j\, F^l_i - F^h_i\, F^l_j)\} + 2v^n\, x_l\, F_{jn}\, F^h_k\, F^l_i = 0. \quad \text{.... (2.26)}$$

Multiplying the above equation by λ_n and using relation (2.2), we obtain the required condition (2.20).

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