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Root Mean Square Radius Of ^{7}Li

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Abstract: The charge root mean square radius is one of the most important static quantity for nuclear structure as well as reaction calculations. The measured value of charge root mean square (RMS) radius of the ground state of 7 lithium nucleus in the present work is calculated using cluster model wave function along with resonating group method (RGM) generator coordinate method (GCM) and complex generator coordinate technique (CGCT). The wavefunction is written by considering alpha deuteron and alpha clusters. Relative motion is very well considered between alpha- deuteron and alpha-neutron clusters. Using CGCT we have transformed this cluster model wavefunction into antisymmetrized product of single particle wavefunction in the form of Slater determinant. The parameter coordinates, generator coordinates and center-of-mass coordinates are used to simplify expressions. The width parameters of deuteron and alpha clusters are chosen in order to get least binding energy and to obtain the results close to the experimental data.

Key Words: cluster model • resonating group method, • complex generator coordinate technique • wavefunction • root mean square radius (RMS)

Introduction

^{7}Li nucleus in ground state is a naturally occurring stable nucleus and its abundance is more than 90%. It is a key component of molten fluoride coolant in molten-salt reactors. An important structural property of the light nucleus ^{7}Li in ground state is calculated in the present paper. High energy electron scattering is one of the most powerful method that can reveal important geometrical parameters pertaining to nuclear structure. One key reason is that the interaction between the electron and the nucleus is understood with a high degree of precision. Since the electron carries a negative charge, it is deflected upon striking the nucleus based on the distribution of charge within the nucleus. Hence these studies give us information about the static distribution of charge and magnetization in nuclei. Because electron- nucleus interaction in the scattering method is relatively weak so the internal structure of the target nucleus is not disturbed significantly. Also, in electron scattering studies (Phillips 1977) we can vary the three-momentum transfer parameters of the nucleus for a fixed energy loss of electron. We can know from this knowledge the details of the spatial distribution of transition charge and current density which is rich and unique information on the structure of nucleus.

Formulation of RMS radius of 7Li nucleus in ground state

In this paper we have used cluster model (Ikeda 2007 and Horuichi 2008) wave function of 'seven lithium nucleus' (Sinha 2021) to calculate the charge root mean square radius (RMS).

^{7}Li nucleus consists of three protons and four neutrons and it is assumed that nucleus is made

of three clusters, alpha cluster composed of two protons and two neutrons, deuteron cluster consists of one proton and one neutron and neutron being considered as a single neutron cluster (Brink 1963 and Sinha 2021).

Generator coordinate method (GCM) and Complex generator coordinate technique (CGCT) are used to construct the microscopic wave function. The nuclear shell model (Khanna 1968 and Stovell 1969), begins with an average potential that resembles the shape of a harmonic oscillator or a square well.

Therefore, a more plausible approach has been employed, involving a complicated potential known as Wood-Saxon potential (Sinha 2020). The cluster model wave function for 7 lithium nucleus in ground state with center of mass wavefunction $Z(\bar{R}_{cm})$ can be written as:

$$\Phi_{\frac{3}{2}M} = A\,[\Phi(\alpha)\,\Phi(d)\,\chi_{l,ml}\,(\bar{R}'_{\alpha} - \bar{r}'_{d})\chi_{l,ml}(\bar{R}'_{\alpha} - \bar{R}'_{7})\,Z(\bar{R}_{cm})\,] \quad (1)$$

Where $J^{\pi=\frac{3^-}{2}}$ is the spin and parity of the nucleus in ground state. 'A' denotes the antisymmetrization operator, while $\Phi(\alpha)$ and $\Phi(d)$ represent the wavefunctions of the alpha and deuteron cluster, χ_{lml} refers to relative motion wavefunction and $Z(\bar{R}_{cm})$ denotes wavefunction of centre of mass.

The wavefunction $\Phi_{\frac{3}{2}M}$ has been transformed into a single-particle wavefunction combined into an antisymmetrized product by using an integral representation for $\Phi_{\frac{3}{2}M}$ leading equation (1), resulting in the following form:

$$\Phi_{\frac{3}{2}M} = A \int \Phi_s\,(\alpha, R'_{\alpha})\delta\,(\bar{R}_{\alpha} - \bar{R}'_{d})\,\Phi_s\,(d, R'_{d})\delta\,(\bar{R}_{d} - \bar{R}'_{d})\,\delta\,(\bar{r}_7 - \bar{r}'_7)\,\xi_{\alpha}\,\xi_{d}\chi_{l,ml}(\bar{R}'_{\alpha} - \bar{R}'_{d})\chi_{l,ml}\,(\bar{R}'_{\alpha} - \bar{r}'_7) \times$$

$$Z_{cm}\left(\frac{4\,\bar{R}'_{\alpha} + 2\bar{R}'_{d} + \bar{r}'_7}{7}\right) dR'_{\alpha}\,dR'_{d}\,dr'_7 \quad (2)$$

Substituting values of all parameters i.e., values of spin, isospin, angular numbers, relative motion wavefunctions and center-of-mass wavefunction, we get the final wavefunction as

$$\Phi_{\frac{3}{2},\frac{3}{2}} = A \int \prod_{J=1}^{4}\prod_{K=5}^{6} \xi_{\alpha}\,\xi_{d}\,\xi_{n}\,\exp\{\frac{-\alpha}{2}(\bar{r}_j - \iota\bar{p})^2\}$$

$$\exp.\left\{\frac{-\alpha}{2}\,(r_K - i_Q)^2\right\}\,\exp.\{\frac{-\alpha}{2}\,(\bar{r}_7 + 4i\bar{p} + 2iQ)^2\}\,\chi_{11}\,(\bar{R}''_1)\,\chi_{10}\,(\bar{R}''_2)$$

$$\exp.\,[-2\alpha\,\{(\bar{P}+\bar{Q})^2 - 2\bar{P}\,.\,\bar{Q}\} + \alpha\,R''^2_1 + \frac{4}{7}\alpha R''^2_2 - 2i\,\alpha\bar{R}''_1.\,(P-Q) - \frac{36i}{7}\,\alpha\,R''_2.\,(P+Q) - 8\alpha\,(P+Q)^2\,d\bar{P}\,d\bar{Q}\,dR''\,dR''_2 \quad (3)$$

Calculation of charge RMS radii

The charge RMS radii of ^{7}Li can be calculated as

$$\sqrt{< R_{ch}{}^2 >} = \sqrt{< R_m{}^2 > + < R_p{}^2 >} \quad (4)$$

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The mean square matter radius $< R_m{}^2 >$ describes the overall distribution of matter, including protons and neutrons, representing the nucleus as a whole (K Okamoto et al., 1974). Additionally, the mean square matter radius of a proton $< R_p{}^2 >$ has a numerical value of 0.64 fm².

Using equation (3) we can write $< R_m{}^2 >$ as

$$< R_m{}^2 > = \frac{1}{7} \frac{< \Phi_{\frac{3}{2},\frac{3}{2}} | \sum_{i=1}^{7} (r_i - \bar{R}cm)^2 |_i{}^2 | \Phi_{\frac{3}{2},\frac{3}{2}} >}{< \Phi_{\frac{3}{2},\frac{3}{2}} | \Phi_{\frac{3}{2},\frac{3}{2}} >} - \frac{1}{3\alpha} \quad (5)$$

In equation (5), $< >'_R$ (bra, ket) shows the integral for both the relative coordinates along with the summation over spin and isospin coordinates. The quantity $\sum_{i=1}^{7} \overline{(r_i} - \bar{R}cm)^2$ may be rewritten using the equation

$$\sum_{i=1}^{7} \overline{(r_i)} = \bar{R}_{cm} \quad (6)$$

$$\sum_{i=1}^{7} \overline{(r_i} - \bar{R}cm)^2 = r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + r_7^2 + R_{cm}^2 - 2\bar{R}_{cm}(\bar{r}_1 + \bar{r}_2 + \bar{r}_3 + \bar{r}_4 + \bar{r}_5 + \bar{r}_6 + \bar{r}_7)$$

$$= r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + r_7^2 - 7R_{cm}^2 \quad (7)$$

substituting eq. (7) is eq. (5) we get

$$< R_m^2 > = \frac{\frac{1}{7} < \Phi_{\frac{3}{2},\frac{3}{2}} | \sum_{i=1}^{7} r_i{}^2 | \Phi_{\frac{3}{2},\frac{3}{2}} >}{< \Phi_{\frac{3}{2},\frac{3}{2}} | \Phi_{\frac{3}{2},\frac{3}{2}} >} - \frac{< \Phi_{\frac{3}{2},\frac{3}{2}} | R_{cm}^2 | \Phi_{\frac{3}{2},\frac{3}{2}} >}{< \Phi_{\frac{3}{2},\frac{3}{2}} | \Phi_{\frac{3}{2},\frac{3}{2}} >} \quad (8)$$

Where the second term can be written and solved as

$$< R_{cm}^2 > = \frac{1}{7} \cdot \frac{3}{2\alpha} \quad (9)$$

Substituting eq. (9) in eq.(8) and then substitute in eq.(4) we get

$$\sqrt{< R_{ch}{}^2 >} = 2.42 \text{ fm}$$

Conclusion

In our theoretical calculation we have varied the width parameters α and β in the range .1 to .5 and found .233 fm^{-1} and .421 fm^{-1} respectively, which gives the best value of binding energy in the ground state. Within this short range it is expected that the binding energy will not change in any significant manner. The present calculated value of charge RMS radius for the ground state of 7 lithium is 2.42 fm. From other studies using Harmonic oscillator (HO) model and Model independent (MI) RMS charge radius comes out to be 2.39 fm and 2.41 fm respectively (Bumiller 1972 and Szalata 1967). According to one more calculation **(**Suelzle 1968**)** using HO shell model the charge RMS radius is 2.39 + 0.03 fm . Our result is in good agreement with the experiment result which is 2.4312fm (Angeli 2004). Hence our presently developed method for writing wave function gives good result of charge RMS radius, we can use the same wavefunction in future to calculate charge form factor and other parameters accurately.

Our present technique the complex generator coordinate technique is highly effective, as it transforms the cluster model wavefunction into an anti-symmetrized product of single-particle wavefunctions. This simplifies the evaluation of matrix elements, which appear in calculations of the normalization constant, rms radius,

quadrupole moment, charge form factor and other similar quantities.

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