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Generalization Of Micro-Open Set And An Application In Healthcare

Mridul Kumar Gupta[1] • Shivangi Tyagi[1]

[1]*Chaudhary Charan Singh University, Meerut, India*

Corresponding Author: shivangimathematics@gmail.com

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Abstract: This study proposes a novel category of micro-semi-open set within a micro-topological space, referred to as micro-S_β (briefly ${}^{\mathfrak{m}}S_\beta$) open set. This set lies between micro-S_p-open set, and micro-semi-open set. Additionally, we developed a soft set model of micro topology to facilitate its application in healthcare. By minimizing attributes in an information system, this method improves efficiency, and supports better decision-making in healthcare. A comparison with the soft set representation of nano-topology is also provided.

Keywords: micro-S_β-open • micro-semi-open • clopen • soft set

1. Introduction

Thivagar and Richard (2013) introduced nano topology, focusing on upper, and lower approximations, and boundary region, involving up to five nano-open sets. Richard (2013) provided an application of nano topology in medical diagnosis in terms of soft set by reducing attributes in an information system. Chandrasekar (2019) extended this concept to micro-topology, exploring micro-open sets, micro-semi-open sets, and other ideas. Subsequently, researchers have identified numerous forms of micro-open sets, many of which have been thoroughly discussed in key papers of Chandrasekar (2019), Ganesan et al. (2021), Ibrahim (2020), Jassim et al. (2021) and Shareef et al. (2021). Maheswari et al. (2023) introduced the concept of micro-S_p-open set, and have studied various characteristics of this set in micro topological spaces. In the present work, we propose a new concept called micro-S_β-open set, which represents a stronger form of micro-semi-open set in micro topological space. An application in healthcare is also provided.

2. Preliminaries

In this paper, the space $(U, \mathfrak{I}_{\mathfrak{R}}(X))$ refers to nano topological space, and the space $(U, \mathfrak{I}_{\mathfrak{R}}(X), \mu_{\mathfrak{R}}(X))$ (or just space U), always means micro-topological space (or briefly ${}^{\mathfrak{m}}$T space) regarding the subset X of space U. Here we utilize the symbol '${}^{\mathfrak{m}}$' to represent the notion 'micro'.

Definition 2.1 Chandrasekar (2019) Consider $(U, \mathfrak{I}_{\mathfrak{R}}(X))$ as a nano topological space , then the set $\mu_{\mathfrak{R}}(X) = \{O \cup (O' \cap \mu)\}: O, O' \in \mathfrak{I}_{\mathfrak{R}}(X)$ and $\mu \notin \mathfrak{I}_{\mathfrak{R}}(X)\}$ is designated as ${}^{\mathfrak{m}}$topology on U concerning X. The trio $(U, \mathfrak{I}_{\mathfrak{R}}(X), \mu_{\mathfrak{R}}(X))$ is termed ${}^{\mathfrak{m}}$T space. The components of $\mu_{\mathfrak{R}}(X)$ are recognized as ${}^{\mathfrak{m}}$open sets, and the supplement of ${}^{\mathfrak{m}}$open set is recognized as a ${}^{\mathfrak{m}}$closed set.

Here, Chandrasekar (2019), Ganesan et al. (2021) and Ibrahim (2020) present the following definition.

Definition 2.2 If U be a ${}^{\mathfrak{m}}$T space, then a subset Q of U is said to be:

(i) ${}^{\mathfrak{m}}$pre-open (resp. ${}^{\mathfrak{m}}$semi-open, ${}^{\mathfrak{m}}\alpha$-open, ${}^{\mathfrak{m}}$b-open, ${}^{\mathfrak{m}}\beta$-open set, ${}^{\mathfrak{m}}$regular-open,) if $Q \subseteq {}^{\mathfrak{m}}int[{}^{\mathfrak{m}}cl(Q)]$ $(Q \subseteq {}^{\mathfrak{m}}cl[{}^{\mathfrak{m}}int(Q)], Q \subseteq {}^{\mathfrak{m}}int[{}^{\mathfrak{m}}cl[{}^{\mathfrak{m}}int(Q)]], Q \subseteq {}^{\mathfrak{m}}cl[{}^{\mathfrak{m}}int(Q)] \cup [{}^{\mathfrak{m}}int[{}^{\mathfrak{m}}cl(Q)]], Q \subseteq {}^{\mathfrak{m}}cl[{}^{\mathfrak{m}}int[{}^{\mathfrak{m}}cl(Q)],$ and $Q = {}^{\mathfrak{m}}int[{}^{\mathfrak{m}}cl(Q)]$ resp).

(ii) $^{\mathfrak{m}}\pi$-open (resp. $^{\mathfrak{m}}S_p$-open, θ- $^{\mathfrak{m}}$open) if Q is the finite union of $^{\mathfrak{m}}$regular-open sets (resp. Q is $^{\mathfrak{m}}$semi-open, and Q = $\bigcup\{P_i : P_i$ is $^{\mathfrak{m}}$pre-closed set$\}$, for each q ∈ Q, there exists a $^{\mathfrak{m}}$open set O such that q ∈ O ⊆ $^{\mathfrak{m}}cl(Q)$ ⊆ Q).

(iii) $^{\mathfrak{m}}$closure of Q is described as $^{\mathfrak{m}}cl(Q) = \bigcap\{H$: H is $^{\mathfrak{m}}$closed such that $Q \subseteq H\}$, and $^{\mathfrak{m}}$interior of a set Q is symbolized as $^{\mathfrak{m}}int(Q)$, and is specified as, $^{\mathfrak{m}}int(Q) = \bigcup\{H$: H is $^{\mathfrak{m}}$open such that $H \subseteq Q\}$.

The families of all $^{\mathfrak{m}}$pre-open, $^{\mathfrak{m}}$semi-open, $^{\mathfrak{m}}\alpha$-open, $^{\mathfrak{m}}$b-open, $^{\mathfrak{m}}\beta$-open set, $^{\mathfrak{m}}$regular open, $^{\mathfrak{m}}\pi$-open, $^{\mathfrak{m}}S_p$-open, and θ- $^{\mathfrak{m}}$open subsets of U are resp. represented by the symbols $\mathcal{MPO}$(U, X), $\mathcal{MSO}$(U, X), $\mathcal{M}\alpha\mathcal{O}$(U, X), $\mathcal{M}b\mathcal{O}$(U, X), $\mathcal{M}\beta\mathcal{O}$(U, X), $\mathcal{MRO}$(U, X), $\mathcal{M}\pi\mathcal{O}$(U, X), $\mathcal{M}S_p\mathcal{O}$(U, X), and $\mathcal{M}\theta\mathcal{O}$(U, X).

Note 1. Some important results obtained are as follows.
(i) Maheswari et al. (2023) In a space U, the union of two $^{\mathfrak{m}}$semi-open sets is again $^{\mathfrak{m}}$semi-open set, and every θ- $^{\mathfrak{m}}$open set is $^{\mathfrak{m}}S_p$-open.
(ii) Ibrahim (2020) In a space U, each $^{\mathfrak{m}}$open (resp. $^{\mathfrak{m}}$semi-open, $^{\mathfrak{m}}$pre-open, $^{\mathfrak{m}}\alpha$-open, $^{\mathfrak{m}}$regular-open) set is $^{\mathfrak{m}}$b-open set as well as $^{\mathfrak{m}}\beta$-open, and the intersection of two $^{\mathfrak{m}}\beta$-closed sets is again a $^{\mathfrak{m}}\beta$-closed set.
(iii) Shareef et al. (2021) A space U is said to be $^{\mathfrak{m}}$locally indiscrete if each $^{\mathfrak{m}}$open set in U is $^{\mathfrak{m}}$closed.

Definition 2.3 Lashin and Medhat (2005) An information system (IS) consists of the form (U, Z,$\{E_z\}$, f_z), where U represents a non-empty finite set of objects known as the universe, Z is a finite non-empty set of attributes, E_z is the set of attribute values for an attribute z ∈ Z, and f_z: U → E_z is the information function. In cases where $E_z(y)$ equals a missing value for a particular y ∈ U, and z ∈ Z, the IS is classified as an incomplete information system (IIS). Otherwise, it is classified as a complete information system (CIS).

Definition 2.4 Molodtsov (1999) Let U represents an initial universal set, and E be a collection of parameters. A pair (S, E) is defined as a soft set over U if and only if S is a function from E into the set containing all subsets of U. That is, S: E → $\wp$(U), where $\wp$(U) represents the power set of U.

3. Micro-S_β-Open Set

In this section, we explore and investigate the idea of $^{\mathfrak{m}}S_\beta$-open set. We analyse and compare this set with different kinds of $^{\mathfrak{m}}$near-open sets.

Definition 3.1 A $^{\mathfrak{m}}$semi-open set Q within a $^{\mathfrak{m}}$T space U is defined as $^{\mathfrak{m}}S_\beta$-open if, for every x ∈ Q, there exists a $^{\mathfrak{m}}\beta$-closed set J such that x ∈ J ⊆ Q. The collection of all $^{\mathfrak{m}}S_\beta$-open sets in U is symbolized as $\mathcal{MS}_\beta\mathcal{O}$(U,X), and the complement of $^{\mathfrak{m}}S_\beta$-open set is referred to as $^{\mathfrak{m}}S_\beta$-closed set. The collection of all $^{\mathfrak{m}}S_\beta$ closed sets in U is represented by $\mathcal{MS}_\beta\mathrm{C}$(U, X).

Proposition 3.2 Given that Q ∈ $\mathcal{MS}_\beta\mathcal{O}$(U, X), it follows that Q ∈ $\mathcal{MSO}$(U, X).

Proof. The proof stems from definition 3.1.

In general, the reverse of Proposition 3.2 does not hold, as exemplified in the following example.

Example 3.3 Consider the universal set U = $\{i_1, i_2, i_3, i_4\}$, with the equivalence relation $U/\mathfrak{R} = \{\{i_1\}, \{i_3\}, \{i_2, i_4\}\}$, and a subset X = $\{i_1, i_2\}$. Then, nano topology is $\mathfrak{I}_\mathfrak{R}$(X) = $\{\phi$, U,$\{i_1\}$, $\{i_1, i_2, i_4\}$, $\{i_2, i_4\}\}$. If we take μ = $\{i_2\}$, then micro topology is defined as, $\mu_\mathfrak{R}$(X) = $\{\phi$, U, $\{i_1\}$, $\{i_2\}$, $\{i_1, i_2\}$, $\{i_1, i_2, i_4\}$, $\{i_2, i_4\}\}$, Here, $\mathcal{MSO}$(U, X) = $\mathcal{M}$b$\mathcal{O}$(U, X) = $\{\phi$, U, $\{i_1\}, \{i_2\}, \{i_1, i_2\}, \{i_1, i_3\}, \{i_2, i_3\}, \{i_2, i_4\}, \{i_1, i_2, i_3\}, \{i_1, i_2, i_4\}, \{i_2, i_3, i_4\}\}$= $\mathcal{M}\beta\mathcal{O}$(U, X), and $\mathcal{MS}_\beta\mathcal{O}$(U, X) = $\{\phi$, U, $\{i_1\}, \{i_1, i_3\}, \{i_2, i_4\}, \{i_1, i_2, i_4\}, \{i_2, i_3, i_4\}$, Clearly, set $\{i_2\}$ ∈ $\mathcal{MSO}$(U, X), but it does not belong to $\mathcal{MS}_\beta\mathcal{O}$(U, X).

Proposition 3.4 A subset Q of a space U is classified as an $^{\mathfrak{m}}S_\beta$-open set if and only if Q is $^{\mathfrak{m}}$semi-open, and can be expressed as the union of $^{\mathfrak{m}}\beta$-closed sets in a space U.

Proof. This conclusion is derived from definition 3.1.

Proposition 3.5 If $\{Q_i : i \in \Delta\}$ represents a family of $^{\mathfrak{m}}S_\beta$-open sets within the space U, then $\bigcup_{i\in\Delta} Q_i$ is a $^{\mathfrak{m}}S_\beta$-open set.

Proof. Suppose $\{Q_i : i \in \Delta\}$ be a collection of $^{\mathfrak{m}}S_\beta$-open sets, then using Proposition 3.2 and part (i) of Note1, for any x ∈ $\bigcup_{i\in\Delta} Q_i \subseteq \mathcal{MSO}$(U, X), there exists a $^{\mathfrak{m}}\beta$-closed set J such that x ∈ J ⊆ $Q_{i_\circ} \subseteq \bigcup_{i\in\Delta} Q_i$, where $i_\circ \in \Delta$. This shows that x ∈ J ⊆ $\bigcup_{i\in\Delta} Q_i$ and, consequently, $\bigcup_{i\in\Delta} Q_i$ is $^{\mathfrak{m}}S_\beta$-open.

In general, the intersection of two $^{\mathfrak{m}}S_\beta$-open sets need not necessarily be $^{\mathfrak{m}}S_\beta$-open, as the following example shows.

Example 3.6 In Example 3.3, if $Q_1 = \{i_1, i_3\}$, and $Q_2 = \{i_2, i_3, i_4\}$,then Q_1, and Q_2 are $^{\mathfrak{m}}S_\beta$-open sets, but $Q_1 \cap Q_2$ is not $^{\mathfrak{m}}S_\beta$-open in U.

Note 2: The collection of $\mathcal{MS}_\beta\mathcal{O}$(U, X) constitutes a supra-topology on U.

Theorem 3.7 Let $Q_1, Q_2 \in \mathcal{MS}_\beta\mathcal{O}$(U, X), and let $\mathcal{MSO}$(U, X) constitute a $^{\mathfrak{m}}$topology on U, then $Q_1 \cap Q_2 \in \mathcal{MS}_\beta\mathcal{O}$(U, X), and $\mathcal{MS}_\beta\mathcal{O}$(U, X) establishes a $^{\mathfrak{m}}$topology on U.

Proof. If $Q_1, Q_2 \in \mathcal{MS}_\beta\mathcal{O}$(U, X), then using Proposition 3.2, $Q_1, Q_2 \in \mathcal{MSO}$(U, X), and since $\mathcal{MSO}$(U, X) constitutes a $^{\mathfrak{m}}$topology, therefore, $Q_1 \cap Q_2 \in \mathcal{MSO}$(U, X). Suppose q ∈ $Q_1 \cap Q_2$, then q ∈ Q_1, and q ∈ Q_2, and thus, there exist $^{\mathfrak{m}}\beta$-closed sets J_1 and J_2 such that q ∈ $J_1 \subseteq Q_1$, and q ∈ $J_2 \subseteq Q_2$, and consequently, q ∈ $J_1 \cap J_2$. Since the intersection of two $^{\mathfrak{m}}\beta$-closed sets is $^{\mathfrak{m}}\beta$-closed (part (ii) of Note 1), therefore $J_1 \cap J_2$ is $^{\mathfrak{m}}\beta$-closed, and hence $Q_1 \cap Q_2$ is $^{\mathfrak{m}}S_\beta$-open. Thus, the collection of $^{\mathfrak{m}}S_\beta$-open sets constitutes a $^{\mathfrak{m}}$topology on U.

Note 3: The notions of $^{\mathfrak{m}}$open sets, and $^{\mathfrak{m}}S_\beta$-open sets are generally independent. For instance, example 3.3 shows that the set $\{i_1, i_2\}$ belongs to $\mu_{\mathfrak{R}}$(X), but it does not belong to $\mathcal{MS}_\beta\mathcal{O}$(U, X), while the set $\{i_2, i_3, i_4\}$ belongs to $\mathcal{MS}_\beta\mathcal{O}$(U, X), however, it does not belong to $\mu_{\mathfrak{R}}$(X).

Theorem 3.8 Suppose Q be any subset in a space U. If Q ∈ $\mathcal{MS}_p\mathcal{O}$(U, X) (resp. Q ∈ $\mathcal{M}\theta\mathcal{O}$(U, X), Q ∈ $\mathcal{MRC}$(U, X), then Q ∈ $\mathcal{MS}_\beta\mathcal{O}$(U, X).

Proof. Using parts (i) and (ii) of Note 1, we get the required result.

The reverse of the statement in Theorem 3.8 is not true universally, as illustrated in Example 3.3. In this instance, the set $\{i_1, i_2, i_4\} \in \mathcal{MS}_{\beta}\mathcal{O}(U, X)$, but it does not belong to $\mathcal{MS}_{p}\mathcal{O}(U, X)$, $\mathcal{M}\theta\mathcal{O}(U, X)$, or $\mathcal{MRC}(U, X)$.

Remark 3.9 Consider Q as a subset of U. If $Q \in \mathcal{MS}_{\beta}\mathcal{O}(U, X)$, and Q is the union of ^{m}pre-closed sets, then $Q \in \mathcal{MS}_{p}\mathcal{O}(U, X)$.

Definition 3.10 Consider a subset Q in a ^{m}T space $(U, \mathfrak{I}_{\mathfrak{R}}(X), \mu_{\mathfrak{R}}(X))$. Then

(i) $^{m}\delta$-closure of Q is defined as, $^{m\delta}cl(Q) = \{w \in U: Q \cap [^{m}int[^{m}cl(H)]] \neq \phi, H \in \mu_{\mathfrak{R}}(X) \text{ and } w \in H\}$.

(ii) Q is $^{m}\delta\beta$-open set if $Q \subseteq {}^{m}cl[^{m}int[^{m\delta}cl(Q)]]$.

The complement of $^{m}\delta\beta$-open sets in U is known as $^{m}\delta\beta$-closed sets, and the collection of all $^{m}\delta\beta$-open sets in U is symbolized as $\mathcal{M}\delta\beta\mathcal{O}(U, X)$.

Remark 3.11 In a space U, each $^{m}\beta$-open set is $^{m}\delta\beta$-open set.

Theorem 3.12 In a space U, each $^{m}S_{\beta}$-open set is ^{m}b-open set, $^{m}\beta$-open set, as well as $^{m}\delta\beta$-open set.

Proof. The proof follows from Proposition 3.2, Part (ii) of Note 1, and Remark 3.11.

The reverse of Theorem 3.12 is generally false, as illustrated in Example 3.3. In this instance, the set $\{i_1, i_2\}$ is ^{m}b-open set, $^{m}\beta$-open set, and $^{m}\delta\beta$ -open set, but it is not $^{m}S_{\beta}$-open set.

Theorem 3.13 If a subset Q of space U is ^{m}clopen, then Q is $^{m}S_{\beta}$-clopen.

Proof. Since Q is ^{m}clopen, therefore, $^{m}int(Q) = {}^{m}cl(Q)$, and $Q \subseteq {}^{m}cl[^{m}int[^{m}cl(Q)]]$, consequently, Q is $^{m}\beta$-open. Since Q is ^{m}clopen, therefore, the supplement of Q (briefly Q^c) is also ^{m}clopen, and so $Q^c \subseteq {}^{m}cl[^{m}int[^{m}cl(Q^c)]]$. Thus, Q^c is $^{m}\beta$-open, and hence Q^c is $^{m}\beta$-closed, and consequently, Q and Q^c are $^{m}\beta$-clopen, and since Q, and Q^c are ^{m}semi-open sets being ^{m}open, therefore, for a ^{m}semi-open set Q, and for each $q \in Q$, there exists a $^{m}\beta$-closed set Q such that $q \in Q \subseteq Q$. Thus, Q is $^{m}S_{\beta}$-open, and hence, using a similar argument, Q^c is $^{m}S_{\beta}$-open.

4. Soft Set Representation of Micro-Topology and its Application

Molodtsov (1999) introduced soft set theory as a mathematical approach to handling uncertainties. Maji and Roy (2002) demonstrated the initial practical use of soft sets in decision-making problems. Recently, Sanabria et al. (2023) used this concept as a tool for vocal risk diagnosis. In this section, we define the representation of a soft set for a ^{m}topology, and employ it to reduce attributes in medical diagnosis.

Definition 4.1 Let U be a non-empty finite universal set, and $\mu_{\mathfrak{R}}(X)$ represents ^{m}topology on U concerning a non-empty subset X of U. Let E = U and the universe of the soft set $W = \mu_{\mathfrak{R}}(X)$, then the set $S_U = \{(y, S(y)): y \in U \text{ and } S : U \longrightarrow \wp(\mu_{\mathfrak{R}}(X))\}$ where $S(y) = \{O \in \mu_{\mathfrak{R}}(X) : y \in O, \forall\, y \in U\}$, is referred to as the representation of soft set for $\mu_{\mathfrak{R}}(X)$.

Example 4.2 Let $U = \{i_1, i_2, i_3, i_4, i_5\}$, $U/_{\mathfrak{R}} = \{\{i_1, i_2\}, \{i_3\}, \{i_4, i_5\}\}$ and $X = \{i_3, i_4\}$ then $\mathfrak{I}_{\mathfrak{R}}(X) = \{\phi, U, \{i_3\}, \{i_3, i_4, i_5\}, \{i_4, i_5\}\}$. If $\mu = \{i_4\}$ then $\mu_{\mathfrak{R}}(X) = \{\phi, U, \{i_4\}, \{i_3\}, \{i_3, i_4\}, \{i_3, i_4, i_5\}, \{i_4, i_5\}\}$ is ^{m}topology on U with respect to X. The set $S_U = \{(i_1, \{U\}), (i_2, \{U\}), (i_3, \{U\}, \{i_3\}, \{i_3, i_4\}, \{i_3, i_4, i_5\}), (i_4, \{U\}, \{i_4\}, \{i_3, i_4\}, \{i_3, i_4, i_5\}, \{i_4, i_5\}), (i_5, \{U, \{i_3, i_4, i_5\}, \{i_4, i_5\}\})\}$ is the representation of soft set of $\mu_{\mathfrak{R}}(X)$.

Remark 4.3 Pawlak (1982) An information system contains multiple attributes, and selecting the minimal attributes for object classification is crucial. A key question is whether a subset of attributes can fully describe the database knowledge. An IS may have multiple reducts, but researchers often focus on a particular reduct, like the minimal reduct or one including key attributes, known as the core. The core represents the essential subset of attributes, as removing any would affect classification ability. So, here we define this core in terms of a soft set as follows.

Definition 4.4 Consider an information system (U, Z), where the attribute set Z is categorized into two classes – 'CA' of condition attributes, and 'DA' of decision attributes. The notion $S_U(CA)$ denotes the representation of the soft set for the ^{m}topology $\mu_{\mathfrak{R}}(X)$ on U where $X \subseteq U$. If a subset Q of CA satisfies $S_U(CA) = S_U(Q)$ and $S_U(CA) \neq S_U(M - \{x\})$ for each $x \in Q$, then it is considered the core of Z.

Algorithm:

Step 1 Let U represents a finite universal set and let Z represents a finite set of attributes that is categorized into two groups – CA for condition attributes and DA for decision attributes. Use this information to create a data table.

Step 2: Determine the family $U/(\mathfrak{R} = CA)$ of equivalence classes corresponding to CA. Next, calculate the approximations $\mathcal{U}_{CA}(X)$ (upper approximation), $\mathcal{L}_{CA}(X)$ (lower approximation), and boundary region $\mathcal{B}_{CA}(X)$ for a non-empty subset X of U.

Step 3: Find ^{m}topology $\mu_{CA}(X)$ and the soft set representation $S_U(CA)$ of $\mu_{CA}(X)$.

Step 4: Eliminate the attribute y from CA and then determine the approximations and boundary region of X in relation to $CA - \{y\}$.

Step 5: Formulate ^{m}topology $\mu_{CA-\{y\}}(X)$, and the soft set model $S_U(CA - \{y\})$.

Step 6: Steps 4 and 5 should be repeated for all attributes in CA.

Step 7: Set $Q = \{y \in CA\colon\ S_U(CA - \{y\}) \neq\ S_U(CA)\}$and repeat steps 4 and 5 for each $y \in Q$.

Step 8: Continue with step 7 until CORE (Z) is achieved.

Table 1: Patients' details with HIV symptoms

Patient ID	**Fever (F)**	**Swollen lymph nodes (S)**	**Night sweats (N)**	**Weight loss(W)**	**HIV Status (HIV)**
Pt^1	Yes	No	No	No	Negative
Pt^2	No	Yes	No	Yes	Positive
Pt^3	Yes	No	No	Yes	Positive
Pt^4	No	No	Yes	No	Negative
Pt^5	Yes	Yes	No	No	Positive
Pt^6	No	Yes	Yes	Yes	Negative
Pt^7	Yes	No	Yes	Yes	Positive
Pt^8	No	No	Yes	No	Positive

Table 1 contains details of eight patients who visited a doctor with one or other symptoms of HIV, such as fever, swollen lymph nodes, night sweats, and weight loss. In the table, "Yes" signifies the presence of the symptom, while "No" indicates its absence. The columns display the symptoms of patients with HIV, whereas

the rows represent individual patients. Each cell in the table contained an attribute value. The last column indicates whether the patient is HIV-positive based on the presence of symptoms.
Here, U = $\{Pt^1, Pt^2, Pt^3, Pt^4, Pt^5, Pt^6, Pt^7, Pt^8\}$, the set of patients, and Z = $\{F, S, N, W, HIV\}$, the set of attributes which is categorized into two groups, CA = $\{F, S, N, W\}$, condition attributes, and DA = $\{HIV\}$, decision attribute. The set of equivalence classes corresponding to CA is defined by U/CA = $\{\{Pt^1\}, \{Pt^2\}, \{Pt^3\}, \{Pt^4, Pt^8\}, \{Pt^5\}, \{Pt^6\}, \{Pt^7\}\}$.

Case 1: (HIV Positive Patients)
The set of patients who tested positive for HIV is represented by set X = $\{Pt^2, Pt^3, Pt^5, Pt^7, Pt^8\}$. Then $\mathcal{L}_{CA}$(X) = $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\mathcal{U}_{CA}$(X) = $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, and $\mathcal{B}_{CA}$(X) = $\{Pt^4, Pt^8\}$. Hence, nano topology $\mathfrak{I}_{CA}$(X) = {U, ϕ, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\{Pt^4, Pt^8\}$. If μ = $\{Pt^3\} \notin \mathfrak{I}_{CA}$(X), then μ_{CA}(X) = {U, ϕ, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\{Pt^4, Pt^8\}$, $\{Pt^3, Pt^4, Pt^8\}$, $\{Pt^3\}$}. Therefore, the soft set formulation of μ_{CA}(X) is presented by $S_U(CA)$ = {(Pt^1, {U}), (Pt^2, {U}, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$}), ($Pt^3$, {U, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\{Pt^3, Pt^4, Pt^8\}$, $\{Pt^3\}$}), (Pt^4, {U, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\{Pt^4, Pt^8\}$, $\{Pt^3, Pt^4, Pt^8\}$}), (Pt^5, {U, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$}), ($Pt^6$, {U}), ($Pt^7$, {U, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$}), ($Pt^8$, {U, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\{Pt^4, Pt^8\}$, $\{Pt^3, Pt^4, Pt^8\}$})}.
Step 1: When the attribute "fever (F)" is excluded from CA, we get U/(CA-{F}) ={$\{Pt^1\}, \{Pt^2\}, \{Pt^3\}, \{Pt^5\}, \{Pt^6\}, \{Pt^7\}, \{Pt^4, Pt^8\}$}, therefore, corresponding upper, and lower approximations, and boundary region are provided by $\mathcal{L}_{CA-\{F\}}$(X) = $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\mathcal{U}_{CA-\{F\}}$(X) = $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$and $\mathcal{B}_{CA-\{F\}}$(X) = $\{Pt^4, Pt^8\}$, which are the same as $\mathcal{L}_{CA}$(X), $\mathcal{U}_{CA}$(X), and $\mathcal{B}_{CA}$(X), and hence, $S_U(CA-\{F\}) = S_U(CA)$. On removal of "weight loss (W)" from CA, U/(CA$-\{W\}$) = {$\{Pt^1, Pt^3\}, \{Pt^2\}, \{Pt^5\}, \{Pt^6\}, \{Pt^7\}, \{Pt^4, Pt^8\}$}, and $\mathcal{L}_{CA-\{W\}}$(X) =$\{Pt^2, Pt^5, Pt^7\}$, $\mathcal{U}_{CA-\{W\}}$(X) = $\{Pt^1, Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\mathcal{B}_{CA-\{W\}}$(X) = $\{Pt^1, Pt^3, Pt^4, Pt^8\}$, which are different from $\mathcal{L}_{CA}$(X), $\mathcal{U}_{CA}$(X), and $\mathcal{B}_{CA}$(X), and hence using a similar process as in cast 1, we get $S_U(CA-\{W\}) \neq S_U(CA)$.
Similarly, when "night sweats (N)" is excluded from CA, U/(CA-{N}) = {$\{Pt^1\}, \{Pt^5\}, \{Pt^2, Pt^6\}, \{Pt^3, Pt^7\}, \{Pt^4, Pt^8\}$}, and $\mathcal{L}_{CA-\{N\}}$(X) = $\{Pt^3, Pt^5, Pt^7\}$, $\mathcal{U}_{(CA-\{N\})}$(X) = $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^6, Pt^7, Pt^8\}$, $\mathcal{B}_{(CA-\{N\})}$(X) = $\{Pt^2, Pt^4, Pt^6, Pt^8\}$, which are different from $\mathcal{L}_{CA}$(X), $\mathcal{U}_{CA}$(X), and $\mathcal{B}_{CA}$(X), and hence using a similar argument as in cast 1, we get $S_U(CA-\{N\}) \neq S_U(CA)$. When "swollen lymph nodes" is excluded from CA, U/(CA$-\{S\}$) = {$\{Pt^2\}, \{Pt^1, Pt^5\}, \{Pt^6\}, \{Pt^3\}, \{Pt^7\}, \{Pt^4, Pt^8\}$}, and $\mathcal{L}_{(CA-\{S\})}$(X) = $\{Pt^2, Pt^3, Pt^7\}$, $\mathcal{U}_{(CA-\{S\})}$(X) = $\{Pt^1, Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\mathcal{B}_{(CA-\{S\})}$(X) = $\{Pt^1, Pt^4, Pt^5, Pt^8\}$, which are different from $\mathcal{L}_{CA}$(X), $\mathcal{U}_{CA}$(X), and $\mathcal{B}_{CA}$(X). Hence using a similar argument as in cast 1, we get S_U(CA-{S}) $\neq S_U(CA)$.
Step 2: Let Q = {W, N, S} = CA$-\{F\}$, then $S_U(Q) = S_U(CA)$. Consider Q $-\{W\} = \{N, S\}$ for which U/(Q $-\{W\}$) ={$\{Pt^1, Pt^3\}, \{Pt^2, Pt^5\}, \{Pt^4, Pt^7, Pt^8\}, \{Pt^6\}$}, and $\mathcal{L}_{Q-\{W\}}$(X) = $\{Pt^2, Pt^5\}$, $\mathcal{U}_{Q-\{W\}}$(X) =$\{Pt^1, Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\mathcal{B}_{Q-\{W\}}$(X) = $\{Pt^1, Pt^3, Pt^4, Pt^7, Pt^8\}$.Therefore, using a similar argument as in case 1, we get $S_U(Q-\{W\}) \neq S_U(CA)$. Similarly, U/(Q $-\{N\}$) ={$\{Pt^1, Pt^4, Pt^8\}, \{Pt^3, Pt^7\}, \{Pt^2, Pt^6\}, \{Pt^5\}$}, and $\mathcal{L}_{Q-\{N\}}$(X) = $\{Pt^3, Pt^5, Pt^7\}$, $\mathcal{U}_{Q-\{N\}}$(X) = U, $\mathcal{B}_{Q-\{N\}}$(X) = $\{Pt^1, Pt^2, Pt^4, Pt^6, Pt^8\}$. Hence,

using similar argument as in case 1, we get $S_U(Q - \{N\}) \neq S_U(CA)$. Also, U/(Q – $\{S\}$) = $\{\{Pt^1, Pt^5\}, \{Pt^2, Pt^3\}, \{Pt^4, Pt^8\}, \{Pt^6, Pt^7\}\}$, and hence $\mathcal{L}_{Q-\{S\}}(X) = \{Pt^2, Pt^3\}$, $\mathcal{U}_{Q-\{S\}}(X) = U$, $\mathcal{B}_{Q-\{S\}}(X) = \{Pt^1, Pt^3, Pt^4, Pt^7, Pt^8\}$. Now, using a similar argument as in case 1, we get $S_U(Q-\{S\}) \neq S_U(CA)$. Hence, CORE (Z) = $\{W, N, S\}$.

Case 2: (HIV negative patients) The set of patients who tested negative for HIV is represented by the set Y = $\{Pt^1, Pt^4, Pt^6\}$. Then, for any $\mu \notin \mathfrak{I}_{CA}(Y)$, we follow the same procedures as in case 1, and achieve identical results. That is, CORE (Z) = {W, N, S}.

Now, we will implement the above application with a soft set model of nano topology, which was given by Richard (2013), and then compare it with our model. We are not going to include the algorithm here, as it can be found in the work of Richard (2013).

From Table 1. we have U = $\{Pt^1, Pt^2, Pt^3, Pt^4, Pt^5, Pt^6, Pt^7, Pt^8\}$, the set of patients, and Z = $\{F, S, N, W, HIV\}$, the set of attributes, which is categorized into two groups, CA = $\{F, S, N, W\}$, condition attributes, and DA = $\{HIV\}$, decision attribute. The set of equivalence classes corresponding to CA is defined by U/CA = $\{\{Pt^1\}, \{Pt^2\}, \{Pt^3\}, \{Pt^4, Pt^8\}, \{Pt^5\}, \{Pt^6\}, \{Pt^7\}\}$.

Case 1: (HIV Positive Patients)

The set of patients who tested positive for HIV is represented by set X = $\{Pt^2, Pt^3, Pt^5, Pt^7, Pt^8\}$. Then $\mathcal{L}_{CA}(X) = \{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\mathcal{U}_{CA}(X) = \{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, and $\mathcal{B}_{CA}(X) = \{Pt^4, Pt^8\}$.Hence, the nano topology $\mathfrak{I}_{CA}(X)$ = {U, ϕ, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$,$\{Pt^4, Pt^8\}$}. Now, the soft set formulation of $\mathfrak{I}_{CA}(X)$ is given by, $S_U(CA)$ = {(Pt^1,{U}), (Pt^2,{ U, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$}), ($Pt^8$, {U, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\{Pt^4, Pt^8\}$}), (Pt^3, {U, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$}), ($Pt^4$,{U, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, Pt^4, Pt^8}}), (Pt^5,{U,$\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$}), ($Pt^6$, {U}), ($Pt^7$, {U, $\{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$})).

Step 1: When the attribute "fever (F)" is excluded from CA, we get U/(CA-{F}) = $\{\{Pt^1\}, \{Pt^2\}, \{Pt^3\}, \{Pt^5\}, \{Pt^6\}, \{Pt^7\}, \{Pt^4, Pt^8\}\}$, therefore, corresponding upper, and lower approximations, and boundary region are given by $\mathcal{L}_{CA-\{F\}}(X) = \{Pt^2, Pt^3, Pt^5, Pt^7\}$, $\mathcal{U}_{CA-\{F\}}(X) = \{Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, and $\mathcal{B}_{CA-\{F\}}(X) = \{Pt^4, Pt^8\}$, which are same as $\mathcal{L}_{CA}(X)$, $\mathcal{U}_{CA}(X)$, and $\mathcal{B}_{CA}(X)$. Hence, $S_U(CA-\{F\}) = S_U(CA)$.

On removal of "weight loss (W)" from CA, U/(CA$-\{W\}$) = $\{\{Pt^1, Pt^3\}, \{Pt^2\}, \{Pt^5\}, \{Pt^6\}, \{Pt^7\}, \{Pt^4, Pt^8\}\}$, and $\mathcal{L}_{CA-\{W\}}(X) = \{Pt^2, Pt^5, Pt^7\}$, $\mathcal{U}_{CA-\{W\}}(X) = \{Pt^1, Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\mathcal{B}_{CA-\{W\}}(X) = \{Pt^1, Pt^3, Pt^4, Pt^8\}$, which are different from $\mathcal{L}_{CA}(X)$, $\mathcal{U}_{CA}(X)$, and $\mathcal{B}_{CA}(X)$, and hence using a similar argument as in cast 1, we get $S_U(CA-\{W\}) \neq S_U(CA)$.

Similarly, when "night sweats (N)" is excluded from CA, U/(CA-{N}) = $\{Pt^1\}, \{Pt^5\}, \{Pt^2, Pt^6\}, \{Pt^3, Pt^7\}, \{Pt^4, Pt^8\}\}$, and $\mathcal{L}_{CA-\{N\}}(X) = \{Pt^3, Pt^5, Pt^7\}$, $\mathcal{U}_{(CA-\{N\})}(X) = \{Pt^2, Pt^3, Pt^4, Pt^5, Pt^6, Pt^7, Pt^8\}$, $\mathcal{B}_{(CA-\{N\})}(X) = \{Pt^2, Pt^4, Pt^6, Pt^8\}$, which are different from $\mathcal{L}_{CA}(X)$, $\mathcal{U}_{CA}(X)$, and $\mathcal{B}_{CA}(X)$, and hence using a similar argument as in cast 1, we get $S_U(CA-\{N\}) \neq S_U(CA)$. When "swollen lymph nodes" is excluded from CA, U/(CA$-\{S\}$) = $\{\{Pt^2\}, \{Pt^1, Pt^5\}, \{Pt^6\}, \{Pt^3\}, \{Pt^7\}, \{Pt^4, Pt^8\}\}$,and $\mathcal{L}_{(CA-\{S\})}(X) = \{Pt^2, Pt^3, Pt^7\}$, $\mathcal{U}_{(CA-\{S\})}(X) = \{Pt^1, Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\mathcal{B}_{(CA-\{S\})}(X) = \{Pt^1, Pt^4, Pt^5, Pt^8\}$, which are different from $\mathcal{L}_{CA}(X)$, $\mathcal{U}_{CA}(X)$, and $\mathcal{B}_{CA}(X)$. Hence using a similar argument as in cast 1, $S_U(CA-\{S\}) \neq S_U(CA)$.

Step 2: Let Q = {W, N, S} = CA$-\{F\}$, then S_U(Q) = $S_U(CA)$. Consider Q $-\{W\}$ = $\{N, S\}$ for which U/(Q - $\{W\}$) = $\{\{Pt^1, Pt^3\}, \{Pt^2, Pt^5\}, \{Pt^4, Pt^7, Pt^8\}, \{Pt^6\}\}$, and $\mathcal{L}_{Q-\{W\}}$(X) = $\{Pt^2, Pt^5\}$, $\mathcal{U}_{Q-\{W\}}$(X) = $\{Pt^1, Pt^2, Pt^3, Pt^4, Pt^5, Pt^7, Pt^8\}$, $\mathcal{B}_{Q-\{W\}}$(X) = $\{Pt^1, Pt^3, Pt^4, Pt^7, Pt^8\}$. Therefore, again using a similar argument as in case 1, we get S_U(Q $-\{W\}$) $\neq S_U(CA)$. Similarly, U/(Q $-\{N\}$) = $\{\{Pt^1, Pt^4, Pt^8\}, \{Pt^3, Pt^7\}, \{Pt^2, Pt^6\}, \{Pt^5\}\}$, and $\mathcal{L}_{Q-\{N\}}$(X) =$\{Pt^3, Pt^5, Pt^7\}$, $\mathcal{U}_{Q-\{N\}}$(X) = U, $\mathcal{B}_{Q-\{N\}}$(X) =$\{Pt^1, Pt^2, Pt^4\ Pt^6, Pt^8\}$. Therefore, using a similar argument as in case 1, S_U(Q $-\{N\}$) $\neq S_U(CA)$. Also, U/(Q $-$ $\{S\}$) = $\{\{Pt^1, Pt^5\}, \{Pt^2, Pt^3\}, \{Pt^4, Pt^8\}, \{Pt^6, Pt^7\}\}$, and hence $\mathcal{L}_{Q-\{S\}}$(X) = $\{Pt^2, Pt^3\}$, $\mathcal{U}_{Q-\{S\}}$(X) = U, $\mathcal{B}_{Q-\{S\}}$(X) = $\{Pt^1, Pt^3, Pt^4, Pt^7, Pt^8\}$. Again, using a similar argument as in case 1, S_U(Q$-\{S\}$) $\neq S_U(CA)$. Hence, CORE (Z) = $\{W, N, S\}$.

Case 2: (**HIV negative patients)** The set of patients who tested negative for HIV is represented by the set Y = $\{Pt^1, Pt^4, Pt^6\}$. Then, we follow the same procedures as in case 1, and achieve the identical results. That is, CORE (Z) = $\{W, N, S\}$.

Observation

From the CORE, we observe that "swollen lymph nodes," "night sweats," and "weight loss" are the primary symptoms linked to HIV that receive greater focus than other symptoms. In this application, it is noteworthy that the outcomes derived from the utilization of soft sets within micro topology, exhibit equivalence to those generated through the application of soft sets within the nano topology.

Discussion: This paper presents $^{\mathfrak{m}}S_{\beta}$-open set, a novel category of $^{\mathfrak{m}}$semi-open set within a $^{\mathfrak{m}}$T space. This set lies between $^{\mathfrak{m}}S_p$ -open set, and $^{\mathfrak{m}}$semi-open sets Additionally, an application in healthcare is presented using soft set model of $^{\mathfrak{m}}$topology, and then a comparison is made with the soft set model of nano-topology. These findings not only deepen the theoretical understanding of micro topology but also demonstrate its transformative potential in real-world applications.

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