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An Efficient Ratio Estimator of Population Mean for Simple Random Sampling

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Abstract: In this paper, a modified exponential type of ratio estimator has been propounded for estimating population mean of a main variable, using Kappa technique under Simple random Sampling (SRS) Scheme to enhance the efficiency of ratio estimator. The expressions of Bias and Mean Square Error (MSE) for the introduced estimator have been derived for approximation up to the order one. The minimum MSE value for the suggested estimator is also attained for the optimum value of the two kappa constants. In addition, both theoretically and empirically, the proposed estimator has been studied and a comparison with the competing estimators. The empirical study has been carried out with both real and simulated datasets to validate the efficiency conditions. The numerical study is performed using R programming and has been found that the introduced estimator is stronger than all other existing estimators of population mean.

Keywords: Ratio Estimator • Auxiliary Variable • Percent Relative Efficient • Mean Square Error • Simple Random Sampling.

Introduction

In the study of samples, the aim of supplementary information supplied by auxiliary variable X is to enhance the estimation of unknown population parameters. The most appropriate estimator for estimating any parameter is the coinciding statistic. Generally, for population mean $\bar{Y}$ of main variable Y, sample mean $\bar{y}$ is the most appropriate unbiased estimator, but has considerably large sampling variance. Our target is to produce an estimator which can be biased but having small variance using auxiliary information as compared to sample mean. As a conclusion of literature review, the information on X benefits the estimators by improving its efficiencies for estimating any parameter as it has a high correlation with Y. When a high positive correlation between X and Y is observed, then ratio estimator is preferred while product estimator is suggested in case of highly negatively correlated X and Y. Otherwise, regression estimators are used for efficiently estimating $\bar{Y}$.

Ratio technique for estimating the parameters is one of the most common and simplest methods of estimation. Cochran (1940) made use of positive correlated X to devise the typical ratio estimator. After Cochran(1940), many researchers for example, Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh *et al.* (2004), Al-Omari (2009), Yan and Tian (2010), Subramani and Kumarpandiyan (2012), Jeelani *et al.* (2013), Yadav *et al.* (2019) have revised the usual ratio estimator using information on X such as Coefficient of Variation C_x, Coefficient of Skewness β_1 and Kurtosis β_2, Median M_x, Quartile Deviation etc. to likely obtain the minimal MSE. Bahl and Tuteja (1991) recommended exponential type ratio and product estimators.To escalate the precision of the usual ratio estimator, Jerajuddin and Kishun (2016) used sample size n instead of X.To enhance estimation, Singh and Tailor (2003) employed information that was already known about the correlation coefficient ρ_{yx} between Y and X while Upadhyaya and Singh (1999) used a transformed X. Under SRS and stratified random sampling methods, Shabbir and Gupta (2011) and Singh and Solanki (2012) suggested better ratio type estimators of $\bar{Y}$ using information on X in quantitative and qualitative formats. Yadav and Kadilar (2013a, 2013b), Sharma and Singh (2013) introduced improved ratio and product type estimators of $\bar{Y}$ using known information on the parameters of X, whereas Yadav and Mishra (2015) and Abid *et al.* (2016) suggested elevated ratio type estimators of $\bar{Y}$ using known information on median

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of Y and some traditional and non-traditional auxiliary parameters. Yadav and Pandey (2017) and Yadav *et al.* (2017) proposed various auxiliary information-based enhanced estimators. Ijaz and Ali (2018), Yadav *et al.* (2018), and Zatezalo *et al.* (2018) introduced elevated ratio and regression type estimators of $\overline{Y}$ using known usual and non-usual location parameters. For improved estimation of $\overline{Y}$, Yadav *et al.* (2019) and Zaman (2019) utilised available information on usual and non-usual properties of X.

Baghel and Yadav (2020) suggested a naive estimator for enhanced estimation of $\overline{Y}$ using known auxiliary parameters while Yadav *et al.* (2021) worked on a new class of estimators for $\overline{Y}$ using regression and ratio exponential estimators. Yadav *et al.* (2022) suggested an elevated estimator for estimating average peppermint production with known X as area of the field. Ali *et al.* (2023) worked on efficient estimation of $\overline{Y}$ utilizing known information on parameters of X.

In this study, the objective and the rationale is to search for more efficient estimator of population mean, which closely estimate the population mean of the investigating variable and enhance the efficiency of an exponential type of ratio estimator compared to other existing estimators under consideration of study. Let the population under study of size *N*. Using SRS scheme, the observations on the required sample size n is taken from (*x*, *y*).

Notations used in the manuscript are as follows:
N: Population size,
n: Sample size,
${}^{N}C_n$: Number of possible cases of n from N,
y: Study Variable,
x: Auxiliary Variable,
M_y, M_x: Medians of populations of y and x respectively
$\overline{X}$, $\overline{Y}$: Population means of auxiliary and study variables,
$\bar{x}$, $\bar{y}$: Sample means of auxiliary and study variables,
ρ_{yx}: Correlation Coefficient between *y* and *x*,
β: Regression Coefficient of *y* on *x*,
S_y^2, S_x^2: Population Mean Square, S_{yx}: Covariance between *y* and *x*,
C_y, C_x: Coefficients of Variation for *y* and *x*,
B(·): Bias of the estimator; *V*(·): Variance of the estimator,
Q_1: First Quartile; Q_3 : Third Quartile,
Q_r: Quartile Range; Q_a: Quartile Average,
QD: Quartile Deviation; *TM*: Tri-Mean,
β_1 : Coefficient of Skewness; β_2: Coefficient of Kurtosis,
MSE(·): Mean Squared Error of the Estimator,
PRE(·): Percentage Relative Efficiency of the estimator to SRS mean

Review of Existing Estimators

In this section, various estimators of $\overline{Y}$ along with their variances or MSEs and constants are presented. The sample mean $\bar{y}$ and various modified ratio estimators of $\overline{Y}$ using known information on X by different authors are given in Table 1. The Variance/MSE of $\bar{y}$ and considered existing estimators as well as their constants are also given in Table 1.

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Table 1: Literature review of various existing estimators of $\bar{Y}$ and their MSE along with constants.

S. No.	Estimators	Variance/MSE	Constants
1.	$t_0 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ Sample Mean	$\lambda\bar{Y}^2 C_y^2$	-
2.	$t_1 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$ Cochran (1940)	$\lambda\bar{Y}^2\left(C_y^2 + C_x^2 - 2C_{yx}\right)$	-
3.	$t_2 = \bar{y}\,exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$ Bahl and Tuteja (1991)	$\lambda\bar{Y}^2\left(C_y^2 + \frac{C_x^2}{4} - C_{yx}\right)$	-
4.	$t_3 = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ Sisodia and Dwivedi (1981)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_{yx}\right)$	$\theta_3 = \frac{\bar{X}}{\bar{X}+C_x}$
5.	$t_4 = \bar{y}\left(\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right)$ Upadhyaya and Singh (1999)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_{yx}\right)$	$\theta_4 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_2}$
6.	$t_5 = \bar{y}\left(\frac{\bar{X}\beta_2+C_x}{\bar{x}\beta_2+C_x}\right)$ Upadhyaya and Singh (1999)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_{yx}\right)$	$\theta_5 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+C_x}$
7.	$t_6 = \bar{y}\left(\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right)$ Singh and Tailor (2003)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 C_{yx}\right)$	$\theta_6 = \frac{\bar{X}}{\bar{X}+\rho_{yx}}$
8.	$t_7 = \bar{y}\left(\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right)$ Singh *et al.* (2004)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_7^2 C_x^2 - 2\theta_7 C_{yx}\right)$	$\theta_7 = \frac{\bar{X}}{\bar{X}+\beta_2}$
9.	$t_8 = \bar{y}\left(\frac{\bar{X}+Q_1}{\bar{x}+Q_1}\right)$ $t_9 = \bar{y}\left(\frac{\bar{X}+Q_3}{\bar{x}+Q_3}\right)$ Al-Omari et al. (2009)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_8^2 C_x^2 - 2\theta_8 C_{yx}\right)$ $\lambda\bar{Y}^2\left(C_y^2 + \theta_9^2 C_x^2 - 2\theta_9 C_{yx}\right)$	$\theta_8 = \frac{\bar{X}}{\bar{X}+Q_1}$ $\theta_9 = \frac{\bar{X}}{\bar{X}+Q_3}$
10.	$t_{10} = \bar{y}\left(\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right)$ Yan and Tian (2010)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_{10}^2 C_x^2 - 2\theta_{10} C_{yx}\right)$	$\theta_{10} = \frac{\bar{X}}{\bar{X}+\beta_1}$
11.	$t_{11} = \bar{y}\left(\frac{\bar{X}\beta_1+\beta_2}{\bar{x}\beta_1+\beta_2}\right)$ Yan and Tian (2010)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_{11}^2 C_x^2 - 2\theta_{11} C_{yx}\right)$	$\theta_{11} = \frac{\bar{X}\beta_1}{\bar{X}\beta_1+\beta_2}$
12.	$t_{12} = \bar{y}\left(\frac{\bar{X}C_x+\beta_1}{\bar{x}C_x+\beta_1}\right)$ Yan and Tian (2010)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_{12}^2 C_x^2 - 2\theta_{12} C_{yx}\right)$	$\theta_{11} = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_1}$
13.	$t_{13} = \bar{y}\left(\frac{\bar{X}+M_x}{\bar{x}+M_x}\right)$ Subramani and Kumarpandiyan (2012a)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_{13}^2 C_x^2 - 2\theta_{13} C_{yx}\right)$	$\theta_7 = \frac{\bar{X}}{\bar{X}+M_x}$
14.	$t_{14} = \bar{y}\left(\frac{\bar{X}C_x+M_x}{\bar{x}C_x+M_x}\right)$ Subramani and Kumarpandiyan (2012a)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_{14}^2 C_x^2 - 2\theta_{14} C_{yx}\right)$	$\theta_{14} = \frac{\bar{X}C_x}{\bar{X}C_x+M_x}$
15.	$t_{15} = \bar{y}\left(\frac{\bar{X}+Q_r}{\bar{x}+Q_r}\right)$ Subramani and Kumarpandiyan (2012b)	$\lambda\bar{Y}^2\left(C_y^2 + \theta_{15}^2 C_x^2 - 2\theta_{15} C_{yx}\right)$	$\theta_{15} = \frac{\bar{X}}{\bar{X}+Q_r}$
16.	$t_{16} = \bar{y}\left(\frac{\bar{X}+QD}{\bar{x}+QD}\right)$	$\lambda\bar{Y}^2\left(C_y^2 + \theta_{16}^2 C_x^2 - 2\theta_{16} C_{yx}\right)$	$\theta_{16} = \frac{\bar{X}}{\bar{X}+QD}$

	Subramani and Kumarpandiyan (2012b)		
17.	$t_{17} = \bar{y}\left(\frac{\bar{X}+Q_a}{\bar{x}+Q_a}\right)$ Subramani and Kumarpandiyan (2012b)	$\lambda\bar{Y}^2(C_y^2+\theta_{17}^2C_x^2-2\theta_{17}C_{yx})$	$\theta_{17}=\frac{\bar{X}}{\bar{X}+Q_a}$
18.	$t_{18} = \bar{y}\left(\frac{\bar{X}\beta_1+QD}{\bar{x}\beta_1+QD}\right)$ Jeelani*et al.*(2013)	$\lambda\bar{Y}^2(C_y^2+\theta_{18}^2C_x^2-2\theta_{18}C_{yx})$	$\theta_{18}=\frac{\bar{X}\beta_1}{\bar{X}\beta_1+QD}$
19.	$t_{19} = \bar{y}\left(\frac{\bar{X}+n}{\bar{x}+n}\right)$ Jerajuddin and Kishun (2016)	$\lambda\bar{Y}^2(C_y^2+\theta_{19}^2C_x^2-2\theta_{19}C_{yx})$	$\theta_{19}=\frac{\bar{X}}{\bar{X}+n}$
20.	$t_{20} = \bar{y}\left(\frac{a.b\bar{X}+c.d}{a.b\bar{x}+c.d}\right)$ $t_{20(1)} = \bar{y}\left(\frac{\beta_2M_x\bar{X}+\rho_{yx}}{\beta_2M_x\bar{x}+\rho_{yx}}\right)$ $t_{20(2)} = \bar{y}\left(\frac{\beta_2M_x\bar{X}+\rho_{yx}C_x}{\beta_2M_x\bar{x}+\rho_{yx}C_x}\right)$ $t_{20(3)} = \bar{y}\left(\frac{\beta_1M_x\bar{X}+\rho_{yx}}{\beta_1M_x\bar{x}+\rho_{yx}}\right)$ $t_{20(4)} = \bar{y}\left(\frac{\beta_1M_x\bar{X}+\rho_{yx}C_x}{\beta_1M_x\bar{x}+\rho_{yx}C_x}\right)$ Yadav *et al.* (2019)	$\lambda\bar{Y}^2(C_y^2+\theta_{20}^2C_x^2-2\theta_{20}C_{yx})$ $\lambda\bar{Y}^2(C_y^2+\theta_{20(1)}^2C_x^2-2\theta_{20(1)}C_{yx})$ $\lambda\bar{Y}^2(C_y^2+\theta_{20(2)}^2C_x^2-2\theta_{20(2)}C_{yx})$ $\lambda\bar{Y}^2(C_y^2+\theta_{20(3)}^2C_x^2-2\theta_{20(3)}C_{yx})$ $\lambda\bar{Y}^2(C_y^2+\theta_{20(4)}^2C_x^2-2\theta_{20(4)}C_{yx})$	$\theta_{20}=\frac{a.b\bar{X}}{a.b\bar{X}+c.d}$ $\theta_{20(1)}=\frac{\beta_2M_x\bar{X}}{\beta_2M_x\bar{X}+\rho_{yx}}$ $\theta_{20(2)}=\frac{\beta_2M_x\bar{X}}{\beta_2M_x\bar{X}+\rho_{yx}C_x}$ $\theta_{20(3)}=\frac{\beta_1M_x\bar{X}}{\beta_1M_x\bar{X}+\rho_{yx}}$ $\theta_{20(4)}=\frac{\beta_1M_x\bar{X}}{\beta_1M_x\bar{X}+\rho_{yx}C_x}$
21.	$t_{20(5)} = \bar{y}\left(\frac{n\bar{X}+\rho_{yx}}{n\bar{x}+\rho_{yx}}\right)$ $t_{20(6)} = \bar{y}\left(\frac{n\bar{X}+C_x}{n\bar{x}+C_x}\right)$ $t_{20(7)} = \bar{y}\left(\frac{n\bar{X}+\rho_{yx}C_x}{n\bar{x}+\rho_{yx}C_x}\right)$ $t_{20(8)} = \bar{y}\left(\frac{n\rho_{yx}\bar{X}+C_x}{n\rho_{yx}\bar{x}+C_x}\right)$ $t_{20(9)} = \bar{y}\left(\frac{nC_x\bar{X}+\rho_{yx}}{nC_x\bar{x}+\rho_{yx}}\right)$ Yadav *et al.* (2019)	$\lambda\bar{Y}^2(C_y^2+\theta_{20(5)}^2C_x^2-2\theta_{20(5)}C_{yx})$ $\lambda\bar{Y}^2(C_y^2+\theta_{20(6)}^2C_x^2-2\theta_{20(6)}C_{yx})$ $\lambda\bar{Y}^2(C_y^2+\theta_{20(7)}^2C_x^2-2\theta_{20(7)}C_{yx})$ $\lambda\bar{Y}^2(C_y^2+\theta_{20(8)}^2C_x^2-2\theta_{20(8)}C_{yx})$ $\lambda\bar{Y}^2(C_y^2+\theta_{20(9)}^2C_x^2-2\theta_{20(9)}C_{yx})$	$\theta_{20(5)}=\frac{n\bar{X}}{n\bar{X}+\rho_{yx}}$ $\theta_{20(6)}=\frac{n\bar{X}}{n\bar{X}+C_x}$ $\theta_{20(7)}=\frac{n\bar{X}}{n\bar{X}+\rho_{yx}C_x}$ $\theta_{20(8)}=\frac{n\rho_{yx}\bar{X}}{n\rho_{yx}\bar{X}+C_x}$ $\theta_{20(9)}=\frac{nC_x\bar{X}}{nC_x\bar{X}+\rho_{yx}}$
22.	$t_{21} = \alpha\bar{y}+(1-\alpha)\bar{y}\left(\frac{ab\bar{X}+c.d}{ab\bar{x}+c.d}\right)$ Baghel and Yadav (2020)	$\lambda\bar{Y}^2\left(C_y^2-\frac{C_{yx}}{C_x^2}^2\right)$	-

Suggested Estimator

Searls (1964) has shown that a constant multiple of $\bar{y}$ is more efficient than $\bar{y}$ while estimating $\bar{Y}$. Motivated by Searls (1964), we devise an improved exponential-type ratio estimator for the estimation of $\bar{Y}$, motivated by Baghel and Yadav (2020), as,

$$t_P = \kappa_1\bar{y} + \kappa_2\bar{y}\exp\left[\frac{(ab\bar{X}+cd)-(ab\bar{x}+cd)}{(ab\bar{X}+cd)+(ab\bar{x}+cd)}\right] \quad (1)$$

Where *a*, *b*, *c*, and *d* are the auxiliary parameters and κ_1 and κ_2 are the constants which minimizes the MSE of suggested estimator.

To acquire the Bias and MSE of the introduced estimator, we have the following approximations:

$\bar{y} = \bar{Y}(1+e_0)$ & $\bar{x} = \bar{X}(1+e_1)$

such that $E(e_0) = E(e_1) = 0$ and $E(e_0^2) = \lambda C_y^2$, $E(e_1^2) = \lambda C_x^2$, $E(e_0, e_1) = \lambda C_{yx}$

where, $\lambda = \frac{1}{n} - \frac{1}{N}$ & $C_{yx} = \rho_{yx} C_y C_x$

Now we put the proposed estimator in terms of e_i's as,

$$t_P = \kappa_1 \bar{Y}(1+e_0) + \kappa_2 \bar{Y}(1+e_0) \exp\left[\frac{(ab\bar{X}+cd) - (ab\bar{X}(1+e_1)+cd)}{(ab\bar{X}+cd) + (ab\bar{X}(1+e_1)+cd)}\right]$$

$$= \kappa_1 \bar{Y}(1+e_0) + \kappa_2 \bar{Y}(1+e_0) \exp\left[\frac{-\theta e_1/2}{1+\theta e_1/2}\right], \quad \text{where, } \theta = \frac{ab\bar{X}}{ab\bar{X}+cd}$$

$$= \kappa_1 \bar{Y}(1+e_0) + \kappa_2 \bar{Y}(1+e_0) \exp\left[\frac{-\theta e_1}{2}\left(1+\frac{\theta e_1}{2}\right)^{-1}\right]$$

Elaborating the expression $\left(1+\frac{\theta e_1}{2}\right)^{-1}$, simplifying and confining terms up to the first order of approximation, we get

$$t_p = \kappa_1 \bar{Y}(1+e_0) + \kappa_2 \bar{Y}(1+e_0)\left(1 - \frac{\theta e_1}{2} + \frac{3\theta^2 e_1^2}{8}\right) \tag{2}$$

Now subtracting $\bar{Y}$ on both sides of the above equation (2), we obtain

$$t_p - \bar{Y} = \bar{Y}\left[\kappa_1(1+e_0) + \kappa_2\left(1 + e_0 - \frac{\theta e_1}{2} + \frac{3\theta^2 e_1^2}{8} - \frac{\theta e_0 e_1}{2}\right) - 1\right] \tag{3}$$

On both sides, taking the expectation of (3), we possess the Bias of the suggested estimator of approximation up to order one after putting the different expectations values as,

$$B(t_p) = \bar{Y}\left[\kappa_1 + \kappa_2\left(1 + \frac{3\theta^2 \lambda C_x^2}{8} - \frac{\theta \lambda C_{yx}}{2}\right) - 1\right] \tag{4}$$

Squaring and taking expectations on both sides of (3), we obtain the MSE of the introduced estimator for an approximation of first order after putting the different expectations values as,

$$MSE(t_p) = \bar{Y}^2\left[\kappa_1^2(1+\lambda C_y^2) + \kappa_2^2(1+\lambda C_y^2 + \theta^2\lambda C_x^2 - 2\theta\lambda C_{yx}) - 2\kappa_1 - 2\kappa_2\left(1 + \frac{3\theta^2\lambda C_x^2}{8} - \frac{\theta\lambda C_{yx}}{2}\right) + 2\kappa_1\kappa_2\left(1 + \lambda C_y^2 + \frac{3\theta^2\lambda C_x^2}{8} - \theta\lambda C_{yx}\right) + 1\right]$$

$$= \bar{Y}^2\left[A\kappa_1^2 + B\kappa_2^2 - 2\kappa_1 - 2C\kappa_2 + 2D\kappa_1\kappa_2 + 1\right] \tag{5}$$

where, $A = (1+\lambda C_y^2)$

$$B = (1 + \lambda C_y^2 + \theta^2 \lambda C_x^2 - 2\theta\lambda C_{yx})$$

$$C = \left(1 + \frac{3\theta^2\lambda C_x^2}{8} - \frac{\theta\lambda C_{yx}}{2}\right)$$

$$D = \left(1 + \lambda C_y^2 + \frac{3\theta^2\lambda C_x^2}{8} - \theta\lambda C_{yx}\right)$$

The mean square error in equation (5) is minimum for,

$\kappa_1 = \frac{(B-CD)}{(AB-D^2)}$ & $\kappa_2 = \frac{(AC-D)}{(AB-D^2)}$

and the minimum MSE is,

$$MSE_{min}(t_p) = \bar{Y}^2\left[1 + \frac{P}{Q^2}\right] \tag{6}$$

where,

$$P = A(B-CD)^2 + B(AC-D)^2 - 2(B-CD)(AB-D^2) - 2C(AC-D)(AB-D^2) + 2D(B-CD)(AC-D),$$

$$Q = (AB - D^2)$$

Theoretical Efficiency Comparison

In this section, the efficiency comparison of the introduced estimator t_p has been carried out with the competing estimators of $\bar{Y}$ and the efficiency conditions for introduced estimator to be better than the competing estimators have been obtained. The introduced estimator outperforms the mentioned estimators under the following conditions, resulting from the comparison of MSE with other existing estimators are given in the Table 2.

Table 2: Theoretical comparison & Efficiency Conditions

S. No.	Estimators	Conditions
1.	$MSE_{min}(t_p) < V(t_0)$	$\frac{P}{Q^2} < \lambda C_y^2 - 1$
2.	$MSE_{min}(t_p) < MSE(t_1)$	$\frac{P}{Q^2} < \lambda(C_y^2 + C_x^2) - (1 + 2\lambda C_{yx})$
3.	$MSE_{min}(t_p) < MSE(t_2)$	$\frac{P}{Q^2} < \lambda(C_y^2 + \frac{C_x^2}{4}) - (1 + \lambda C_{yx})$
4.	$MSE_{min}(t_p) < MSE(t_i)$	$\frac{P}{Q^2} < \lambda(C_y^2 + \theta_i^2 C_x^2) - (1 + 2\lambda\theta_i C_{yx}); i = 3, \ldots.., 20$
5.	$MSE_{min}(t_p) < MSE(t_{21})$	$\frac{P}{Q^2} < \lambda[C_y^2 + C_x^2(\alpha^2\theta_{21}^2 - 2\alpha\theta_{21}^2 + \theta_{21}^2) - 2\lambda C_{yx}(2\theta_{21} - 2\alpha\theta_{21}) - 1]$

Numerical Study

To verify the theoretical finding, we have taken into account, the two data sets from real life problems. For the data sets under consideration, the values of various parameters have been calculated and the MSEs of different competing estimators and the suggested estimators have been calculated. The percentages Relative Efficiency (PRE) of various estimators over the $\bar{y}$ estimator have also been calculated. The PRE for different estimators over $\bar{y}$ estimator have been calculated by using the following formula:

$$PRE = \frac{V(t_0)}{MSE(\cdot)} \times 100$$

Details of Data set 1:

Source: Daroga Singh and F.S. Chaudhary (2020, Page-176)

Auxiliary Variable(x): Area under guava orchards (in acres) during 1971-72

Study Variable(y): Total of guava trees during 1971-72

Table 3: Population Parameters of Data 1

N=13	n=3	$\bar{Y} = 746.9231$	$\bar{X} = 5.661538$
$S_y = 450.2417$	$S_x = 3.679984$	$C_y = 0.6027953$	$C_x = 0.6499971$
$M_y = 714$	$M_x = 5.99$	$\rho_{yx} = 0.9005962$	$C_{yx} = 0.3528673$
$\beta_1 = 0.7045741$	$\beta_2 = 6.456733$	$f = 0.2307692$	$\lambda = 0.2564103$
${}^N C_n = 286$	$Q_1 = 2.200$	$Q_3 = 6.580$	$Q_r = 4.38$
$Q_a = 4.923333$	QD=2.19	TM=5.19	

Table 4: MSE & PRE of various existing estimators & suggested estimators for Data 1

Estimators	Variance/MSE	PRE	Estimators	Variance/MSE	PRE
t_0	51978.87	100	t_{16}	10607.32	490.0282
t_1	11461.69	453.5007	t_{17}	15271.41	340.3672

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t_2	16610.78	312.9225	t_{18}	11993.37	433.3968
t_3	10051.14	517.1438	t_{19}	11812.35	440.0383
t_4	23293.99	223.1428	$t_{20(1)}$	11381.11	456.7117
t_5	11132.11	466.9273	$t_{20(2)}$	11409.01	455.5949
t_6	9866.095	526.8433	$t_{20(3)}$	10817.86	480.4912
t_7	18005.2	288.6881	$t_{20(4)}$	11019.88	471.6826
t_8	10620.02	489.442	$t_{20(5)}$	10611.84	489.8195
t_9	18215.8	285.3504	$t_{20(6)}$	10809.54	480.861
t_{10}	9997.225	519.9314	$t_{20(7)}$	10865.08	478.4032
t_{11}	22240.1	233.7169	$t_{20(8)}$	10750.11	483.5195
t_{12}	9821.202	529.2516	$t_{20(9)}$	10303.01	504.5019
t_{13}	17193.89	302.3102	t_{21}	9820.181	529.3066
t_{14}	22312.5	232.9585	$\boldsymbol{min(t_p)}$	**9767.706**	**532.1502**
t_{15}	14271.41	364.2168			

Details of Data set 2:
Source: Daroga Singh and F.S. Chaudhary (2020, Page-177)
Auxiliary Variable(*x*): Cultivated Area under wheat in a region during 1973
Study Variable(*y*): Area under wheat in a region during year 1974
Table 5: Population Parameters of Data 2

***N*=34**	***n*=5**	$\bar{Y} = 199.4412$	$\bar{X} = 208.8824$
$S_y = 150.215$	$S_x = 150.506$	$C_y = 0.7531797$	$C_x = 0.7205298$
$M_y = 142.5$	$M_x = 150$	$\rho_{yx} = 0.9800867$	$C_{yx} = 0.5318817$
$\beta_1 = 0.8732281$	$\beta_2 = 5.912272$	$f = 0.1470588$	$\lambda = 0.1705882$
${}^{N}C_n =$**278256**	$Q_1 = 94.25$	$Q_3 = 254.75$	$Q_r = 160.5$
$Q_a = 166.3333$	***QD*=80.25**	***TM*=162.25**	

Table 6: MSE & PRE of various existing estimators & suggested estimators for Data 2

Estimators	Variance/MSE	PRE	Estimators	Variance/MSE	PRE
t_0	3849.248	100	t_{16}	473.1776	813.4891
t_1	153.8905	2501.29	t_{17}	922.6805	417.181
t_2	1120.88	343.413	t_{18}	535.4868	718.8315
t_3	154.5255	2491.011	t_{19}	159.8507	2408.027
t_4	165.4474	2326.57	$t_{20(1)}$	153.8915	2501.275
t_5	153.9924	2499.635	$t_{20(2)}$	153.8912	2501.279
t_6	154.7734	2487.021	$t_{20(3)}$	153.8967	2501.189
t_7	161.3104	2386.237	$t_{20(4)}$	153.895	2501.217
t_8	548.1055	702.2823	$t_{20(5)}$	154.0555	2498.612
t_9	1312.292	293.3224	$t_{20(6)}$	154.0112	2499.33
t_{10}	154.6701	2488.682	$t_{20(7)}$	154.0088	2499.369
t_{11}	162.7818	2364.667	$t_{20(8)}$	154.0137	2499.289
t_{12}	155.0034	2483.331	$t_{20(9)}$	154.121	2497.549

t_{13}	841.4363	457.4616	t_{21}	151.7764	2536.131
t_{14}	1117.772	344.368	$\boldsymbol{min(t_p)}$	**133.1**	**2891.996**
t_{15}	893.9771	430.5757			

Simulation Study

Additionally, a simulated population was created using the R program to analyse the results of competing estimators and the proposed estimator for the unreal population and see the differences between the two. To generate or simulate this population, the parameters (mean vectors and variance-covariance matrix) from the same real populations represented in the numerical study are used to create a realistic data representation. Here, the mean vector represents expected variable values, allowing the simulation to be based on typical observations for effective estimation. The variance-covariance matrix captures data variability and inter-variable relationships, simulating complex interactions common in real data sets. Using a bivariate normal distribution with mean vectors and a variance-covariance matrix, the population for both datasets is created to enhance the simulation's realism as follows:

For Dataset 1:

• Means of [Y, X] as $\mu = [746.9231, 5.661538]$

• Variances and covariance of [Y, X] as $\sigma^2 = \begin{bmatrix} 202717.588 & 1492.182 \\ 1492.182 & 13.54228 \end{bmatrix}$

• Correlation $\rho_{yx} = 0.9032775$

For Dataset 2:

• Means of [Y, X] as $\mu = [199.4412, 208.8824]$

• Variances and covariance of [Y, X] as $\sigma^2 = \begin{bmatrix} 22564.55 & 22158.05 \\ 22158.05 & 22652.06 \end{bmatrix}$

• Correlation $\rho_{yx} = 0.9793408$

Procedure for simulating the population as follows:

(a) With the help of R program, simulated populations for a bivariate normal distribution of Y and X of size N as 5000 have been generated from above parameters.

(b) For this simulated population, parameters have been determined.

(c) Then a sample of size n has been selected from this population using SRSWOR.

(d) Sample statistics and the proposed and existing estimator t_i values are estimated for this sample.

(e) Repeating the steps c) and d) for b=50000 times.

(f) The MSE of every estimator t_i is computed by:

$$MSE(t_i) = \frac{1}{b}\sum_{j=1}^{b}\left(t_{ij} - \overline{Y}\right)^2$$

(g) The PRE of each of the estimator t_i has been calculated using the formula:

$$PRE(t_i) = \frac{MSE(t_c)}{MSE(t_i)} \times 100$$

Table 7: MSE & PRE of mentioned estimators & suggested estimators for simulated population of Data 1 (for sample sizes of n as 40, 50 and 60 respectively)

	Variance/MSE			**PRE**		
Estimators	**n=40**	**n=50**	**n=60**	**n=40**	**n=50**	**n=60**
t_0	5093.5484	4066.6233	3382.0066	100	100	100
t_1	1109.0960	885.4879	736.4159	459.2523	459.2523	459.2523
t_2	1610.7632	1286.0126	1069.5121	316.2196	316.2196	316.2196

t_3	967.2663	772.2529	642.2440	526.5921	526.5921	526.5921
t_4	1419.3344	1133.1783	942.4075	358.8688	358.8688	358.8688
t_5	1043.3048	832.9611	692.7319	488.2129	488.2129	488.2129
t_6	949.6172	758.1621	630.5254	536.3791	536.3791	536.3791
t_7	1143.5586	913.0024	759.2983	445.4121	445.4121	445.4121
t_8	1161.2823	927.1528	771.0665	438.6141	438.6141	438.6141
t_9	2019.2264	1612.1243	1340.7229	252.2525	252.2525	252.2525
t_{10}	1093.0439	872.6722	725.7577	465.9967	465.9967	465.9967
t_{11}	4325.4506	3453.3840	2872.0062	117.7576	117.7576	117.7576
t_{12}	1085.1557	866.3743	720.5201	469.3841	469.3841	469.3841
t_{13}	1608.1077	1283.8925	1067.7489	316.7417	316.7417	316.7417
t_{14}	2080.2077	1660.8110	1381.2132	244.8577	244.8577	244.8577
t_{15}	1497.2009	1195.3459	994.1092	340.2047	340.2047	340.2047
t_{16}	1064.4108	849.8119	706.7459	478.5322	478.5322	478.5322
t_{17}	1608.0024	1283.8083	1067.6790	316.7625	316.7625	316.7625
t_{18}	3957.9704	3312.3170	2845.0525	128.6909	122.7728	118.8733
t_{19}	4192.0117	3346.8480	2783.4056	121.5061	121.5061	121.5061
$t_{20(1)}$	1090.9832	871.0269	724.3894	466.8769	466.8769	466.8769
$t_{20(2)}$	1097.0054	875.8350	728.3880	464.3139	464.3139	464.3139
$t_{20(3)}$	1211.5511	967.2867	804.4438	420.4155	420.4155	420.4155
$t_{20(4)}$	1030.1026	822.4206	683.9660	494.4700	494.4700	494.4700
$t_{20(5)}$	1101.2891	880.4855	732.9415	462.5078	461.8615	461.4293
$t_{20(6)}$	1103.3718	881.8232	733.8721	461.6348	461.1608	460.8441
$t_{20(7)}$	1103.9241	882.1776	734.1184	461.4038	460.9756	460.6895
$t_{20(8)}$	1102.7618	881.4313	733.5996	461.8904	461.3659	461.0153
$t_{20(9)}$	1097.3452	877.9457	731.1717	464.1701	463.1976	462.5461
t_{21}	945.0792	754.5390	627.5122	538.9547	538.9547	538.9547
$\boldsymbol{min(t_p)}$	**944.5796**	**754.2204**	**627.2918**	**539.2397**	**539.1823**	**539.1440**

Table 8: MSE & PRE of mentioned estimators & suggested estimators for simulated population of Data 2 (for sample sizes of n as 40,50 and 60 respectively)

	Variance/MSE			PRE		
Estimators	**n=40**	**n=50**	**n=60**	**n=40**	**n=50**	**n=60**
t_0	547.82584	437.37708	363.74457	100	100	100
t_1	22.63476	18.07130	15.02900	2420.28538	2420.28538	2420.28538
t_2	159.35150	127.22418	105.80597	343.78454	343.78454	343.78454
t_3	22.71227	18.13318	15.08046	2412.02639	2412.02639	2412.02639
t_4	23.25410	18.56577	15.44022	2355.82506	2355.82506	2355.82506
t_5	22.65956	18.09110	15.04546	2417.63690	2417.63690	2417.63690
t_6	22.74534	18.15959	15.10242	2408.51880	2408.51880	2408.51880
t_7	23.03363	18.38975	15.29383	2378.37387	2378.37387	2378.37387
t_8	89.50300	71.45804	59.42807	612.07537	612.07537	612.07537
t_9	214.80555	171.49798	142.62627	255.03337	255.03337	255.03337
t_{10}	22.62824	18.06610	15.02467	2420.98278	2420.98278	2420.98278

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t_{11}	56.64354	45.22348	37.61010	967.14610	967.14610	967.14610
t_{12}	22.62553	18.06393	15.02287	2421.27252	2421.27252	2421.27252
t_{13}	161.25024	128.74011	107.06669	339.73644	339.73644	339.73644
t_{14}	210.78960	168.29170	139.95976	259.89225	259.89225	259.89225
t_{15}	153.48711	122.54213	101.91214	356.91977	356.91977	356.91977
t_{16}	82.29720	65.70502	54.64357	665.66767	665.66767	665.66767
t_{17}	160.09292	127.81612	106.29826	342.19241	342.19241	342.19241
t_{18}	38.81420	36.07192	34.53208	1411.40554	1212.51395	1053.35250
t_{19}	719.13678	574.14953	477.49136	76.17825	76.17825	76.17825
$t_{20(1)}$	22.63492	18.07143	15.02910	2420.26862	2420.26862	2420.26862
$t_{20(2)}$	22.63487	18.07139	15.02907	2420.27357	2420.27357	2420.27357
$t_{20(3)}$	22.62752	18.06552	15.02419	2421.06020	2421.06020	2421.06020
$t_{20(4)}$	22.62965	18.06722	15.02560	2420.83246	2420.83246	2420.83246
$t_{20(5)}$	22.63728	18.07291	15.03011	2420.01646	2420.07035	2420.10625
$t_{20(6)}$	22.63657	18.07246	15.02980	2420.09204	2420.13077	2420.15656
$t_{20(7)}$	22.63653	18.07243	15.02978	2420.09605	2420.13396	2420.15923
$t_{20(8)}$	22.63661	18.07248	15.02981	2420.08796	2420.12750	2420.15384
$t_{20(9)}$	22.63834	18.07358	15.03058	2419.90331	2419.97993	2420.03096
t_{21}	22.40148	17.88505	14.87410	2445.48948	2445.48948	2445.48948
$\boldsymbol{min(t_p)}$	**22.06287**	**17.66953**	**14.72518**	**2483.02172**	**2475.31923**	**2470.22143**

Results and Discussion

(I) Review of literature of existing estimators are shown in Table 1. Table 2 consists of the comparison of various competing estimators with the proposed estimator t_p providing the conditions under which t_p is better than other ones. Table 3 and Table 4 consists of data of population parameters of data sets 1 and 2 respectively, with which we have proved the results empirically. Table 5 and Table 6 display the values of MSE and the PRE of several considered estimators as well as proposed class of estimators according to data set 1 and 2 respectively. Table 7 and Table 8 shows the values of MSE and the PRE of introduced and competing estimators for simulated populations for sample of size 40, 50 and 60 according to data set 1 and 2 respectively. From the above tables, it may be observed the suggested estimator outperform the competing estimators.

(II) For real population:

(a) For data set 1, the variance is 51978.87 of sample mean whereas the MSE of all other competing estimators are varying between 23293.99 to 9820.181. In the proposed class of estimators, their MSE lies between 9766.057 to 9815.086. Here the minimal value of MSE of the introduced class is 9767.706, which is smallest among the class of all mentioned existing estimators while the PRE value of suggested class is 532.1502, which is the greatest out of all the mentioned existing estimators of $\bar{Y}$ under SRS procedure.

(b) For data set 2, the variances 3849.248 of sample mean whereas the MSE of all other existing estimators are lying between 1117.772 to 151.7764. In the introduced class of estimators, their MSE lies between 132.4726 to 133.1. Here the MSE value of the proposed class is 133.1, which is minimum among the discussed existing estimators while the PRE value of suggested class is 2891.996, which is the largest out of all the mentioned estimators of population mean under SRS scheme.

(III) For simulated population:

(a) From Table 7 for data set 1, it is evident that the MSE of introduced estimator is less than the other competing estimators for different sample sizes of simulated population. Here, the MSE values of

t_p are 944.5796, 754.2204, 627.2918 for the sample of size 40, 50 and 60. Also, the PRE values of introduced class are greatest among the mentioned competing estimators, i.e., 539.2397, 539.1823, 539.1440 for the sample of size 40, 50 and 60 respectively, showing improvement over competing estimators.

(b) From Table 8 for data set 2, it is evident that the MSE of introduced estimator is less than the other competing estimators for different sample sizes of simulated population. Here, the MSE values of t_p are 22.06287, 17.66953, 14.72518 for the sample of size 40, 50 and 60 respectively. Also, the PRE values of introduced class are greatest among the mentioned competing estimators, i.e., 2483.02172, 2475.31923, 2470.22143 for the sample of size 40, 50 and 60 respectively, which shows better efficiency of the proposed estimator.

Conclusion

In this paper, we have been able to devise an exponential- type ratio estimator that is more efficient than other competing estimators of $\overline{Y}$ under SRS scheme. The Bias and Mean MSE of t_p for an approximation up to order one have been studied. For t_p, the MSE is minimum for the optimum values of κ_1 & κ_2. Both theoretically as well as numerically, it has been concluded that the propounded estimator has outperformed than other existing competing estimators. Through both real and simulated populations, t_p has the least MSE and largest PRE out of all the mentioned competing estimators of $\overline{Y}$, thereby attaining the goal of the study. Hence, the proposed class of estimators t_p can strongly be advised for empirical use in various applied fields such as commerce, medical sciences, engineering, agriculture, economics etc.

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