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# Effect Of Hall Currents on Free Convection and Mass Transfer in MHD Flow of a Dusty Viscoelastic Liquid (Walters Liquid Model - B)

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**Abstract:** This study investigates the influence of Hall currents on free convection and mass transfer in the magnetohydrodynamic (MHD) flow of a dusty viscoelastic liquid, specifically modeled using Walters-B fluid. The flow is considered between two vertical, heated, porous plates, with a magnetic field applied perpendicular to the direction of fluid motion. The effects of various parameters—such as the Hall parameter (m), Hartmann number (M), and porosity parameter (K)—on the velocities of both the liquid and dust particles are explored. Additionally, the behavior of skin friction for both phases is examined. Numerical results are presented through graphs and tables, demonstrating the impact of these physical parameters on the overall dynamics of the system, including heat and mass transfer processes. This analysis provides valuable insights into the behavior of viscoelastic fluids under combined magnetic and convective influences, which have practical applications in engineering and industrial processes.

**Keywords**: Hall current • Magnetohydrodynamics (MHD) • Walters-B fluid • Heat transfer • Mass transfer

## Introduction

This collection of studies spans several decades of research on fluid mechanics, heat transfer, and mass transfer, with particular attention to phenomena such as dusty flows, magnetic fields, and convection in various systems. The articles collectively examined various dimensions of fluid mechanics, particularly focusing on heat transfer and convection phenomena. Gebhart and Pera (1971) explored vertical natural convection flows driven by thermal and mass diffusion, laying the groundwork for understanding buoyancy-driven flows in environmental and industrial contexts. Raptis and Perdikis (1984) examined free convection under magnetic fields, laying a foundation for MHD boundary-layer theories. Raptist and Singh (1985) addressed the impact of rotation on MHD free-convection flows near accelerated vertical plates, with implications for rotating fluid systems. Hossain and Rashid (1987) investigated the Hall effect in hydromagnetic free convection along a porous flat plate, emphasizing its implications for mass transfer. Nanousis (1992) examined thermal-diffusion effects on MHD-free convection and mass transfer in a rotating fluid, emphasizing the interplay between rotation and thermal gradients in astrophysical and geophysical contexts. Renardy (1994) reviewed advanced mathematical frameworks for fluid mechanics, emphasizing the theoretical underpinnings required to model and analyze the intricate behaviors observed in fluid systems Saffman (1995) provided foundational insights into vortex dynamics, emphasizing its relevance across various natural and engineering systems. Bhattacharyya and Pal (1997) presented another perspective by studying unsteady MHD flows between rotating discs, a model applicable to industrial systems with rotating machinery. Hossain et al. (1997) examined MHD forced and free convection along vertical porous plates, highlighting combined buoyancy and magnetic effects on boundary-layer dynamics. Takhar and Jha (1998) introduced Hall and ion-slip currents in MHD flows near impulsively started plates, emphasizing rotational influences on transient dynamics. Singh et al. (2000) developed computational methods for viscoelastic particulate flows using a distributed Lagrange multiplier, advancing simulations in complex fluid systems. Aboeldahab and Elbarbary (2001) extended this understanding to semi-infinite plates, highlighting the role of Hall currents in enhancing or suppressing convection, which is vital for applications involving boundary-layer flows. Elbashbeshy and Bazid (2004) investigated heat transfer over a stretching surface within a porous medium, focusing on internal heat generation and fluid injection or suction. This foundational work is applicable in polymer extrusion and cooling processes. Attia (2005) examined ion slip effects in dusty fluid MHD flows under pressure gradients. Singh et al. (2005) examined fluid flow within scaffolds under bioreactor rotation, highlighting the role of mechanical rotation in tissue engineering. Ghosh and Pop (2006) presented a novel approach to MHD natural convection near a flat plate of finite dimensions, emphasizing boundary-layer phenomena. Alam et al. (2007) introduced thermophoresis and heat generation into hydromagnetic free convection along inclined flat plates, offering solutions for scenarios where temperature gradients induce particle migration in porous media. Saha et al. (2007) studied Hall currents in MHD laminar natural convection along a permeable flat plate, revealing their impact on heat and mass transfer processes. Bég et al. (2009) addresses the Hartmann–Couette flow in porous channels. Both studies delve into secondary effects like ion slip, Joule heating, and viscous interactions, providing robust numerical methods to analyze these phenomena. Rathod and Tanveer (2009) investigated pulsatile flows of a couple of stress fluids in porous media under magnetic influence, demonstrating the significance of periodic body forces in fluid transport mechanisms. Singh and Gorla (2009) incorporated Hall currents, Joule heating, and thermal diffusion into free convection studies, revealing the intricate coupling of electromagnetic and thermal processes. Turns et al. (2009) highlighted innovative teaching methods in fluid mechanics, improving learning outcomes in engineering education. Uwanta et al. (2011) explored viscoelastic fluid flows past vertical plates with heat dissipation, offering models for thermal energy management. Chamkha et al. (2012) investigated radiation effects in mixed convection over a wedge embedded in porous media filled with nanofluids, contributing to advancements in thermal systems. Jha and Ajibade (2012) studied viscous dissipation effects on natural convection between vertical plates with time-periodic boundaries, relevant to oscillatory heat systems. Jha et al. (2012) analyzed steady and unsteady Couette flows in reactive viscous fluids between vertical porous plates, offering critical insights into chemical-reactive fluid systems. Mahmoud and Waheed (2012) analyzed the MHD flow of a micropolar fluid over a stretching surface, accounting for heat generation (absorption) and slip velocity. Bég (2013) explored numerical methods for solving complex multi-physical magnetohydrodynamics (MHD) problems. The study focuses on developing and applying computational techniques to model the interactions between magnetic fields, fluid flow, and other physical phenomena, such as heat and mass transfer. Chand et al. (2013) extended the analysis to oscillating dusty fluids in porous channels under Hall currents and rotational effects, offering insights into particulate-laden flows. Kumar (2013) extended the study of viscoelastic fluids by examining the influence of Hall currents, rotation, chemical reactions, and radiation on oscillatory dusty flows through porous vertical channels, highlighting applications in atmospheric and industrial fluid systems. Choudhury and Purkayastha (2014) investigated viscoelastic effects on convection flow in a vertically rotating channel partially filled with a porous medium. Their study emphasizes how rotation and viscoelastic properties alter flow stability, with implications for fluid transport in rotating machinery and geophysical systems. Hayat et al. (2014) explored Walters' B fluid under convective boundary conditions, contributing to understanding non-Newtonian fluid flows in heat-sensitive environments. Khan et al. (2014) explored unsteady free convection flow in a Walters-B viscoelastic fluid, incorporating heat transfer effects. Their work provides a foundational understanding of viscoelastic fluid dynamics under transient conditions, which is particularly relevant in polymer and biofluid applications. Prakash et al. (2014) focussed on radiation and Dufour effects in unsteady MHD flow near a wavy vertical plate, revealing the impact of varying temperature and mass diffusion in thermally sensitive environments. Purkayastha and Choudhury (2014) investigated Hall currents and thermal radiation in elastic-viscous fluid flow within a rotating porous channel, emphasizing applications in rotating machinery and porous structures. Seth et al. (2014) extended this by examining Hall currents, rotation, and radiation in unsteady MHD natural convection with chemical reactions, offering applications in environmental and industrial fluid flows. Adesanya et al. (2015a, 2015b) examined pulsating flows through vertical porous channels and consider viscous dissipation effects. They also explore time-periodic boundary conditions in natural convection flows between parallel plates, providing insights into transient and oscillatory behaviors in real-world systems. Das et al. (2016) investigated the impact of Hall currents on unsteady magneto-convection and radiative heat transfer in fluid flow past a porous plate. The study incorporates magnetic field effects, thermal radiation, and convective heat transfer to model complex flow dynamics. Siddiqa et al. (2016) investigated temperature-dependent density effects in natural convection over horizontal circular disks, providing models crucial for thermal systems design. Raptis (2017) further extended this by considering thermal radiation effects in MHD flows past vertical plates. Sivaiah and Reddy (2017) explored radiating fluids influenced by Hall currents along inclined porous plates, shedding light on heat and mass transfer in inclined geometries. Srinivasacharya and Jagadeeshwar (2017) analyzed Hall current and Joule heating effects on flows over exponentially stretching sheets, providing critical insights for material manufacturing processes. Zeytounian (2017) discussed contemporary challenges in fluid dynamics, advocating for interdisciplinary approaches to solve complex fluid systems problems. Gupta and Singh (2018) extended the discussion to magnetic field effects on free convection in liquid metals, electrolytes, and ionized gases, critical for energy and metallurgical processes. Singh et al. (2018) analyzed rotating viscoelastic fluids in unsteady MHD convection, offering models for oscillatory flow systems. Gupta and Rana (2022) further analyzed heat flux effects in MHD flows involving ternary hybrid nanofluids, revealing how nanofluid compositions and stretching surfaces can optimize thermal systems. Maurya and Deo (2022) investigated MHD effects on micropolar fluid flow within a porous cylindrical domain, providing insights into fluid-structure interactions in confined geometries. Patil and Kulkarni (2022) studied Eyring-Powell nanofluid flows, incorporating quadratic mixed convection and multiple diffusion effects, offering models for advanced thermal management systems. Yaseen et al. (2022) studied hybrid nanofluids with nonlinear thermal radiation in porous media, providing applications in energy-efficient systems. Bhadauria et al. (2023) explored how different magnetic field modulations and nanoparticle shapes affect nanofluid instability, demonstrating how precise control of magnetic parameters can optimize flow stability in advanced cooling systems. Ali et al. (2024) focussed on the unsteady MHD flow of a Casson hybrid nanofluid, incorporating thermal radiation and nanoparticle properties. Their study sheds light on hybrid nanofluids' enhanced heat transfer capabilities in porous systems. Das et al. (2024a) modeled reactive fluid convection near a vertical pervious plate under intense magnetic forces, Hall current, and thermo-diffusion. This provides insights into complex flow dynamics influenced by chemical reactions and magnetic fields. Das et al. (2024b) examined the interplay of rotational buoyancy, magnetic fields, Hall currents, and infrared radiation in a Casson fluid medium. Their findings are significant for understanding fluid behaviors in electrically conductive and optically active environments. Das et al. (2024c) also delved into dusty fluid dynamics induced by a ramped thermally active plate, offering valuable models for flows involving particulate-laden fluids under magnetic influence. Rajakumar et al. (2024) explored the effects of radiation absorption, Hall, and ion-slip currents in MHD convection with spanwise temperature fluctuations. Their perturbative study provides detailed models for electromagnetic and thermal interactions in fluid systems. Ramesh, Mebarek-Oudina, and Souayeh (2024) presented a comprehensive exploration of fluid dynamics and nanofluids through advanced mathematical modeling techniques. Singh et al. (2024) focussed on computational fluid dynamics for cylindrical tube flows under steady and transient conditions, offering insights for pipeline and heat exchanger designs. Collectively, these studies contribute to a deeper understanding of fluid dynamics in MHD contexts, especially regarding viscoelastic fluids and the effects of Hall currents.

**Formulation of the Problem**

Let us consider the fully developed unsteady dusty flow of an incompressible, slightly conducting, viscoelastic fluid between two heated porous infinite parallel plates (distance 2h apart) under the influence of a uniform very strong magnetic field applied perpendicular to the flow region. We assume the x-axis along the flow in the mid-way of the plates and the y-axis perpendicular to it. Let u be the liquid velocity, and v the particle velocity in the direction of the x-axis. Our analysis is based on the following assumptions:

- The flow is in the direction of the x-axis and is driven by a constant pressure with negligible body force.
- The dust particles are nonconducting, solid, spherical, and equal in size, uniformly and symmetrically distributed in the flow region and their number density No is constant throughout the motion.
- There is no externally applied electric field and the induced magnetic field and induced magnetic field are negligible.
- The interaction between the particles, buoyancy force on the particles, chemical reactions, and radiation between particles have been neglected.
- Initially, when $t \leq 0$, the channel, walls as well as liquid are assumed to be at the same temperature T, and foreign mass is assumed to be present at a low level and it is

uniformly distributed such that it is everywhere $C_o$.

- When t > 0 the temperature of walls is instantaneously raised to $T_w$ and the species concentration is raised to C thereafter both of them are maintained constant and the thermal diffusion effect (Soret effect) is neglected.

Under these assumptions and Boussinesq's approximation with concentrations, the equations governing the flow are:

$$\frac{\partial u}{\partial t} = g\beta(T - T_o) + g\beta^{+}(C - C_o) + \nu\left(1 - K_o\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{KN_o}{\rho}(v - u)$$
$$-\frac{\nu}{k_1}u - \frac{\sigma}{\rho}.\frac{1}{1+m_1^2}B_o^2 u \quad \dots \quad \dots \quad (1)$$
$$m\frac{\partial v}{\partial t} = K(u - v) \quad \dots \quad \dots \quad (2)$$
$$\frac{\partial T}{\partial t} = \frac{K_T}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \quad \dots \quad \dots \quad (3)$$
$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} \quad \dots \quad \dots \quad (4)$$

where $\rho$ is the density, $g$ is the acceleration due to gravity, β is the coefficient of volume expansion, $\beta^*$ is the coefficient of concentration expansion, $K_o$ is the coefficient of visco-elasticity of the liquid, K is Stoke's resistance coefficient, *m* is the mass of a dust particle, is the Kinematic viscosity, σ is the electric conductivity, $B_o$ is the magnetic induction, $m_1$ is the hall parameter, $k_1$ is the permeability of the porous medium, $C_p$ is the specific heat at constant pressure, $K_T$ is the thermal conductivity, D is the concentration diffusivity.

At $t = 0$, The temperature and concentration level changes according to the following laws :

$$T = T_o + (T_W - T_o)(1 - e^{-at}) \quad \dots \quad (5)$$
$$C = C_o + (C_W - C_o)(1 - e^{-at}) \quad \dots \quad (6)$$

Where *a* is the decay factor.

The initial and boundary conditions relevant to the problem are:

$$t = 0 u = 0 = v, \quad T = T_o, \quad y \in (-d, d) t > 0 u = 0 = v, \quad T = T_o, T = T_o + (T_W - T_o)(1 - e^{-at})$$
$$C = C_o + (C_W - C_o)(1 - e^{-at}) for y = - d$$

$$and \quad u = 0 = vT = T_o, \; T = T_o + (T_W - T_o)(1 - e^{-at})$$
$$C = C_o + (C_W - C_o)(1 - e^{-at}) for y = d (7)$$

We introduce the following non-dimensional quantities

$$y^* = \frac{y}{d}, \; u^* = \frac{u}{d}, \; v^* = \frac{v}{d}, t^* = \frac{\nu t}{d^2}, \; a^* = \frac{ad^2}{\nu}$$
$$T^* = \frac{T - T_o}{T_W - T_o}, \quad C^* = \frac{C - C_o}{C_W - T_o}$$

Equation (1), (2), (3) and (4) after dropping the asterisks (*) can be written as

$$\frac{\partial u}{\partial t} = G_r T + G_m C + \left(1 - E\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{L}{W}(v - u) - \frac{u}{k} - \frac{M^2}{1 + m_1^2}u \quad \dots (8)$$
$$W\frac{\partial v}{\partial t} = (u - v) \quad \dots \quad (9)$$
$$\frac{\partial^2 T}{\partial y^2} - P_r\frac{\partial T}{\partial t} = 0 \quad \dots \quad (10)$$

$$\frac{\partial^2 C}{\partial y^2} - S_c \frac{\partial C}{\partial t} = 0 \qquad \ldots \qquad (11)$$

Where

$$k = \frac{k_1}{d^2}(porositypara.),\ \ S_c = \frac{v}{D}(schmdtNo.),\ \ E = \frac{vK_o}{d^2}$$

$$M = B_o d\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}(hartman \quad number) \qquad P_r = \frac{\mu C_p}{K_T}(PrandtlNo.)$$

$$G_r = \frac{g\beta(T_W - T_o)d}{v}(GrashofNumber) G_m = \frac{g\beta^+(C_W - C_o)d}{v}(ModifiedGrashofNumber)$$

$$L = \frac{mN_o}{\rho}(MassconcentrationofdustParticles), W = \frac{vm}{Kd^2}(relaxationtimeparameterofParticles),$$

The initial and boundary condition (7) becomes

$$t = 0u \ = 0 = vT = 0, y \in (-1,1) t > 0u \ = 0 = vT = (1 - e^{-at}), C = (1 - e^{-at}) fory = -1andu = 0 = vT = (1 - e^{-at}), C = (1 - e^{-at}) fory = 1 \qquad \ldots \qquad (12)$$

**Solution of the Problem**

Following the procedure of Singh et al [9], the solutions of equations (8), (9), (10) and (11) are:

$$T(y,t) = 1 - \frac{cosh\ cosh\ \sqrt{aP_r y}}{cosh\ cosh\ \sqrt{aP_r}} e^{-at} (13)$$

$$C(y,t) = 1 - \frac{cosh\ cosh\ \sqrt{aS_c y}}{cosh\ cosh\ \sqrt{aS_c}} e^{-at} (14)$$

$$u(y,t) = K_1\left[1 - \frac{y}{}\right] + K_2\left[1 - \frac{y}{}\right] e^{-at} \qquad \ldots \qquad (15)$$

$$v(y,t) = K_1\left[1 - \frac{y}{}\right] + \frac{K_2}{(1 - aW)}\left[1 - \frac{y}{}\right] e^{-at} \qquad \ldots \qquad (16)$$

Where

$$K_1 = \frac{G_r + G_m}{M_1^2}, K_2 = \frac{1}{(1 + aE)}\left[\frac{G_r}{(aP_r - M_2^2)} + \frac{G_m}{(aS_c - M_2^2)}\right], M_1^2 = \frac{M^2}{(1 + m_1^2)} + \frac{1}{k}, M_2^2 = \frac{1}{(1 + aE)}\left[M_1^2 - a - \frac{aL}{1 - aW}\right]$$

**Skin Friction**

Let $\tau_1$ and $\tau_p$ be the skin friction for liquid and dust particles respectively, then we have

$$\tau_1 = \left[-\frac{\partial u}{\partial y}\right]_{y=1} = K_1 M_1 + K_2 M_2 . e^{-at}$$

$$\tau_p = \left[-\frac{\partial v}{\partial y}\right]_{y=1} = K_1 M_1 + \frac{K_2 M_2}{(1 - aW)} . e^{-at}$$

## Results and Discussion

Liquid and particles velocity profiles are tabulated in table-1 and table-2 and plotted in Fig-1 and Fig-2 having four Graphs at $a$ = 0.2, $S_c$ = 0.6, $P_r$ = 0.71, t = 1, Gr = 5, $L$ = 0.5, $W$ = 0.5, *E* = 1 and following different values of $m_1$ (Hall parameter), M (Hartman Number) and *K* (porosity parameter).

| **Parameter** | $m_1$ | ***M*** | ***K*** |
|---|---|---|---|
| **For Graph-1** | 0.5 | 1 | 1 |
| **For Graph-2** | 1.0 | 1 | 1 |
| **For Graph-3** | 0.5 | 2 | 1 |
| **For Graph-4** | 0.5 | 1 | 2 |

Skin friction values for liquid and particles are tabulated in Table-3 at $a = 0.2$, $S_c = 0.6$, $P_r = 0.71$, $t = 1$, $Gr = 5$, $L = 0.5$, $W = 0.5$, $E = 1$, $y = 1$ and following different values of m (Hall parameter), M (Hartman number) and K (Porosity parameter).

**Table 1:** Values of velocity of liquid *u* at a = 0.2 $S_c$ = 0.6. $P_r$ = 0.71 t = 1. $G_r$ = 10, 0. $G_m$ = 5, L = 0.5. W = 0.5 E = 1 and different values of $m_1$, M & K

| **y** | **Graph-1** | **Graph-2** | **Graph-3** | **Graph-4** |
|---|---|---|---|---|
| -1 | 0 | 0 | 0 | 0 |
| -0.5 | 0.41445 | 0.34980 | 0.37433 | 0.24697 |
| 0 | 0.49642 | 0.41410 | 0.43368 | 0.28102 |
| 0.5 | 0.41445 | 0.34980 | 0.37433 | 0.24697 |
| 1 | 0 | 0 | 0 | 0 |

**Table 2:** Values of the velocity of particles *v* at a = 0.2 $S_c$ = 0.6. $P_r$ = 0.71 t = 1. $G_r$ = 10, 0. $G_m$ = 5, L = 0.5. W = 0.5 E = 1 and different values of $m_1$, M & K

| **y** | **Graph-1** | **Graph-2** | **Graph-3** | **Graph-4** |
|---|---|---|---|---|
| -1 | 0 | 0 | 0 | 0 |
| -0.5 | 0.09422 | -0.00522 | 0.17729 | -0.13958 |
| 0 | 0.07963 | -0.05006 | 0.18566 | -0.22593 |
| 0.5 | 0.09422 | -0.00522 | 0.17729 | -0.13958 |
| 1 | 0 | 0 | 0 | 0 |

**Table 3:** Values of skin frictions $\tau_1$ and $\tau_p$ at a=0.2 $S_c$ = 0.6. $P_r$ = 0.71 t = 1. $G_r$ = 10, 0. $G_m$ = 5, L=0.5. W=0.5 E=1 and different values of $m_1$, M & K

| $m_1$ | M | K | $\tau_1$ | $\tau_p$ |
|---|---|---|---|---|
| 0.5 | 1 | 1 | 1.47259 | 0.55512 |
| 1.0 | 1 | 1 | 1.27065 | 0.26697 |
| 0.5 | 2 | 1 | 1.47068 | 0.84744 |
| 0.5 | 1 | 2 | 0.96792 | -0.11499 |

**Observations**

1. The observation of **Table 1** indicates variations in the velocity *u* of the fluid for different values of the parameters $m_1$, *M*, and *K* while keeping other parameters constant *(a = 0.2, Sc = 0.6, Pr = 0.71, t = 1, Gr =10, Gm = 5, L = 0.5, W = 0.5, E = 1)*. The data show a symmetrical behavior around *y = 0*, where the velocity peaks near y = 0 and decreases symmetrically towards *y = −1* and *y = 1*. For instance, in ***Graph-1***, the velocity reaches a maximum of *0.49642* at *y = 0*, while in ***Graph-2***, it reaches 0.41410 at the same point. Across all graphs, the velocity is zero at y = ±1. highlighting a consistent pattern. The differences in velocity values across the graphs suggest that the parameters $m_1$, *M*, and *K* significantly influence the flow profile, with higher values

resulting in lower velocities across the boundary layer. This observation helps understand the effect of varying these parameters on fluid flow behavior.

2. **Table 2** presents the values of particle velocity $v$ under specific fluid flow conditions *($a = 0.2$, $Sc = 0.6$, $Pr = 0.71$, $t = 1$, $Gr = 10$, $Gm = 5$, $L = 0.5$, $W = 0.5$, $E = 1$)* while varying the parameters $m_1$, $M$, and $K$. The table highlights significant fluctuations in the velocity values depending on these parameters. In **Graph 1**, for example, the particle velocity increases slightly to 0.09422 at $y = -0.5$ and reaches 0.07963 at $y = 0$, showing a small peak. **Graph 3** exhibits a relatively higher velocity, peaking at 0.18566 at $y = 0$. In contrast, **Graphs 2** and **4** show negative velocity values, indicating a reversal in direction, with the minimum velocity being -0.22593 in **Graph 4** at $y = 0$. Across all graphs, the velocity returns to zero at $y = \pm 1$, consistent with the boundary conditions. These observations suggest that $m_1$, $M$, and $K$ changes significantly affect particle velocity, with some combinations leading to directional reversals and more pronounced velocity variations.

3. **Table 3** illustrates the skin friction values ($\tau_1$ for the fluid and $\tau_p$ for the particles) under given conditions *($a = 0.2$, $Sc = 0.6$, $Pr = 0.71$, $t = 1$, $Gr = 10$, $Gm = 5$, $L = 0.5$, $W = 0.5$, $E = 1$)*, while varying parameters $m_1$, $M$, and $K$. The results show that changes in these parameters lead to noticeable variations in skin friction values. For instance, when $m_1 = 0.5$, $M = 1$, and $K = 1$, $\tau_1$ is 1.47259 and $\tau_p$ is 0.55512. As $m_1$ increases to 1.0 while keeping $M$ and $K$ constant, $\tau_1$ decreases to 1.27065, and $\tau_p$ also decreases to 0.26697, indicating that higher values of $m_1$ reduce the skin friction for both the fluid and particles. In contrast, increasing $M$ to 2 while keeping $m_1$ and $K$ constant increases $\tau_p$ significantly to 0.84744, while $\tau_1$ remains almost unchanged. Finally, increasing $K$ to 2 with $m_1 =$ 0.5 and $M = 1$ results in a lower $\tau_1$ of 0.96792, and $\tau_p$ becomes negative at -0.11499, suggesting that larger $K$ values reduce the fluid's resistance and reverse the friction for particles. This analysis highlights the sensitivity of skin friction to varying physical parameters.

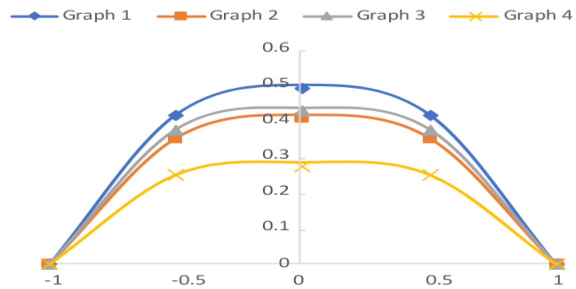


**Fig 1. Graphical Representation of Liquid Velocity Profile**

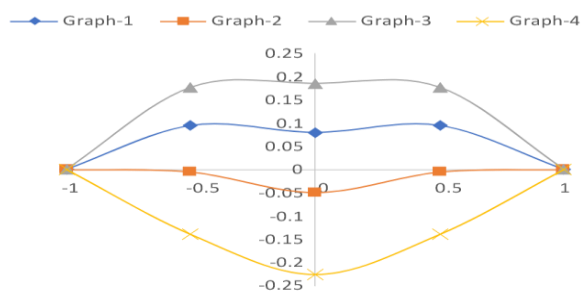


**Fig 2. Graphical Representation of Particles Velocity Profile**

**Concluding Remarks and Future Scope**

In conclusion, this study highlights the significant impact of Hall currents on free convection and mass transfer in magnetohydrodynamic (MHD) flows of a dusty viscoelastic fluid, specifically modeled using the Walters-B liquid model. The investigation reveals that variations in key parameters—such as the Hall parameter (m), Hartmann number (M), and porosity parameter (K)—affect the velocity profiles of both the liquid and dust particles, as well as the skin friction for each phase. Through numerical results presented in tables and graphs, the study demonstrates how increasing values of these parameters generally lead to reductions in velocity and alterations in skin friction, with notable variations depending on the combination of values. This sensitivity to parameter changes offers insights into optimizing fluid flow conditions in practical applications, such as industrial processes involving heat and mass transfer. The study also suggests that future research could explore more complex interactions between these parameters and other physical factors, providing further refinement to the model and its applications in engineering and physics.

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