Shear Free Radiating Fluid Ball Without Horizon

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Received: 12.05.2024; Revised: 20.06.2024; Accepted: 21.06.2024
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Abstract: This paper investigates the novel explicit model of the relativistic field equations for radiating fluid ball in the absence of horizon. The inner metric is without shear, symmetry like sphere and undergoes heat flow in radial direction. The inner metric maintains all appropriate thermodynamic and physical parameters analytically as well as graphically and are matched to the Vaidya external metric across the boundary. Here in the present model, the collapse begins with unlimited mass and radius, and time approaches zero without the event horizon formation. The luminosity is independent of time, and the rate of collapse process will never reach the horizon.

Keywords: radiating fluid ball • exact model • naked singularity • gravitational collapse.

1. Introduction

The gravitational collapse is a theoretical and observable feature in the universe; it captures the focus of general relativity researchers. In relativistic astrophysics the final outcome of the gravitational collapse is an open problem Joshi and Malafarina (2011). As of right now, no established theory exists that can predict this collapse. However, there are many contrary situations where the creation of a naked singularity is most likely to occur.

Constructing a realistically model of a collapsing structure is essential in the context of fundamental gravity in order to fully comprehend the system and the physical behavior of the decreasing structure. Because the laws underlying field equations are highly nonlinear, this is an extremely challenging task to solve them. A number of attempts have been made to make easier things, and different approaches are regularly used to do so. The important contribution is made by Oppenheimer and Snyder (1939), which examined the collapse of an incredibly idealized spherically statuesque particles cloud, was the first step in this direction.

In an effort to comprehend the features and nature of collapsing things, numerous strive have been formed since then to build accurate models of collapsing system. It acquired enormous momentum when Vaidya (1951) offered a solution that combined the modified equations for an adiabatic dispensation of matter introduced by Misner (1965) and Lindquist et al. (1965) with a model defining the external gravitational expansion of a star objects with radiation leaving it.

Collapsing of an object is recognized as a very dissipative form of energy (Herrera and Santos 2004, Herrera et al. 2006, Mitra 2006). On the other hand, two limiting scenarios are outlined where energy dissipation via collapsing fluid distribution occurs. In the first situation, Tewari (1988), Pant et al. (1990), Tewari (1994) discusses a number of radiating matter solutions. Additionally, Pant and Tewari (2011) offered a
quasar structure that was suitable to every extent. As opposed to the second, which is the diffusion case, where dissipation is represented by a heat flow. In this direction Glass (1981) and Santos (1985) with reference to the junction conditions of a radially heated collapsing system are significant. The numerous papers on related field have been published by Banerjee et al. (2002), Maharaj et al. (2005), de Oliveira et al. (1985), Maiti (1987), de Oliveira-Santos (1987), Sarwe-Tikekar (2010), Bonnor et al. (1989), Tewari (2012, 2013), Pinheiro-Chan (2013), Tewari et al. (2014, 2015a, 2015b, 2015c, 2015d), Ivanov (2012), Charan et al. (2021), Kauntey Acharya et al. (2023), Ghosh Avisikta (2024) and references within that describe the energy-radiating collapse of a collapsing fluid. Govender et al. (2020) present realistic analytic models characterized by only two metric components, constrained by spatial development limited through isotropy conditions and the radiating ideal fluid with heat flow is typically explored in order to explain the calculations. In considering the aforementioned considerations, we examine Tewari et al. (2014) solution in an effort to create a realistic radiating ball model. Over the boundary, the Vaidya exterior line element (1953) is matched with the interior line element.

This study is organized into five sections. In section 2, junction circumstances and space-time are discussed. In Section 3 using method of variable separation to separate the space and time variables. The model's accompanying energy conditions are covered in Section 4. In Section 5, the graphical representation and evaluation of the energy conditions are discussed.

2. The matching conditions

The metric element with time like space is given by the following relation

$$ds^2 = -X^2(r,t)dt^2 + Y^2(r,t)(dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2))$$  \hspace{1cm} (1)

For a fluid, the energy-momentum tensor is given as

$$T_{ij} = (\epsilon + p)u_iu_j + pg_{ij} + q_iu_j + q_ju_i$$  \hspace{1cm} (2)

where symbols have their usual meanings.

For fluid distribution (1), the fluid collapse rate is determined by

$$\Theta = \frac{3Y}{XY}$$  \hspace{1cm} (3)

Given (1) and (2), the system of equations that follows

$$\kappa\epsilon = -\frac{1}{Y^2}\left(\frac{2Y''}{Y} - \frac{Y''}{r^2} + \frac{4Y'}{rY} + \frac{3Y^2}{X^2} \right)$$  \hspace{1cm} (4)

$$\kappa p = -\frac{1}{Y^2}\left(\frac{2Y^2}{Y} + \frac{2XY'}{XY} + \frac{2X}{rX} + \frac{2Y'}{rY} \right)$$

$$+ \frac{1}{X^2}\left(-\frac{2Y}{Y} - \frac{\dot{Y}}{Y^2} + \frac{2\ddot{X}}{X} \right)$$  \hspace{1cm} (5)

$$\kappa p = -\frac{1}{Y^2}\left(\frac{2Y'}{Y} + \frac{Y'}{rX} + \frac{X'}{X} + \frac{X'}{rY} \right)$$

$$+ \frac{1}{X^2}\left(\frac{2Y'}{X} - \frac{\dot{Y}}{Y^2} + \frac{2\ddot{X}}{XY} \right)$$  \hspace{1cm} (6)

$$\kappa q = -\frac{2}{XY^2}\left(-\frac{\dot{Y}}{Y} + \frac{X''}{Y^2} + \frac{X'Y'}{XY} \right)$$  \hspace{1cm} (7)

The external metric is given by (1953) as

$$ds^2 = \left(1 - \frac{2M(r)}{R}\right)dv^2 - 2dRdv + R^2(d\theta^2 + \sin^2\theta d\phi^2)$$  \hspace{1cm} (8)
The matching of (1) and (8) at the boundary is given by Santos (1985) as
\[
(rY)_{\Sigma} = R_{\Sigma}v = R(\tau) \tag{9}
\]
\[
(p_{\tau})_{\Sigma} = (qY)_{\Sigma} \tag{10}
\]
\[
m_{\Sigma}(r,t) = M(v) = \left(\frac{r^{3}Y^{2}}{2\Sigma} - r^{2}Y' - \frac{r^{3}Y^{2}}{2Y} \right)_{\Sigma} \tag{11}
\]
This mass at the boundary is given by Cahill et al. (1970) and Misner et al. (1964).

The $Z_{\Sigma}$ and $L_{\Sigma}$ on the boundary are obtained from the following relations
\[
L_{\Sigma} = \frac{1}{2} \left( r^{3}Y^{2}q \right)_{\Sigma} \tag{12}
\]
\[
Z_{\Sigma} = \left[ 1 + \frac{rY'}{Y} + \frac{rY^{2}}{X} \right]^{-1} - 1 \tag{13}
\]

The $L_{\infty}$ at rest is given as
\[
L_{\infty} = \frac{dM}{dv} = \frac{L_{\Sigma}}{(1+Z_{\Sigma})^{2}} \tag{14}
\]

3. Solution of field equations
In equation (1), taking into consideration the following different forms for the metric
\[
X(r,t) = X_{0}(r)g(t) \tag{15}
\]
\[
Y(r,t) = Y_{0}(r)f(t) \tag{16}
\]
With the help of field equations (4), (5), (7), (15) and (16) we have
\[
\epsilon = -\frac{1}{r^{2}Y_{0}} \left( \frac{2Y_{0}'}{Y_{0}} - \frac{Y_{0}'}{r} + \frac{4Y_{0}^{2}}{rY_{0}} \right) + \frac{3f^{2}}{x_{0}g^{2}f^{2}} \tag{17}
\]
\[
p = \frac{1}{r^{2}Y_{0}} \left( \frac{r^{2}Y_{0}'}{rY_{0}} + 2Y_{0}^{2} \frac{Y_{0}'}{x_{0}Y_{0}} + \frac{2X_{0}}{rX_{0}} \right) + \frac{1}{x_{0}g^{2}} \left( -\frac{2f}{f} - \frac{j^{2}}{f^{2}} \right) \tag{18}
\]
\[
q = -\frac{2x_{0}f}{x_{0}g^{2}f^{2}} \tag{19}
\]
The boundary conditions (10), $(p_{\tau})_{\Sigma} = (qY)_{\Sigma}$ now yields at $r = r_{\Sigma} = R_{\Sigma}$ in view of (16), (18) and (19)
\[
\dot{f} + j^{2} - 2\eta \dot{f} = \zeta \tag{20}
\]
where $\eta$ and $\zeta$ are constant and given by
\[
\eta = \left( \frac{\dot{X}_{0}}{X_{0}} \right) \tag{21}
\]
\[
\zeta = \frac{x_{0}}{x_{0}g} \left( \frac{Y_{0}'}{Y_{0}} + 2Y_{0} \frac{Y_{0}'}{x_{0}Y_{0}} + \frac{2X_{0}}{rX_{0}} \frac{X_{0}'}{rX_{0}} \right) \tag{22}
\]
In view of particular solution of above equation is given by
\[
f = -\alpha t \tag{23}
\]
where $\alpha$ is arbitrary constant.

In order for the configuration to collapse we must have $\alpha < 0$, meaning that $\alpha$ must be positive.
\[
\alpha = -\eta + \sqrt{\eta^{2} + \zeta}, \quad \zeta > 0 \tag{24}
\]
The isotropy pressure would given as below
\[
\frac{\dot{x}_{0}}{x_{0}} + \frac{y_{0}}{y_{0}} = \frac{2Y_{0}'}{Y_{0}} \left( \frac{1}{r} \right) \left( \frac{\dot{x}_{0}}{x_{0}} + \frac{y_{0}}{y_{0}} \right) \tag{25}
\]

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The solution of \( (25) \) given by Tewari et al. (2014):

\[
X_0 = d_2 \left( 1 + c_1 r^2 \right)^{\frac{n}{4+8}} + d_1 \left( 1 + c_1 r^2 \right)^{\frac{2-n}{1+1}}
\]

\[
Y_0 = c_2 \left( 1 + c_1 r^2 \right)^{\frac{1}{4+8}}
\]

\[
n = \frac{1}{2} \left( l + 3 \right) \pm \left( l^2 + 10l + 17 \right)^{\frac{1}{2}}
\]

(26) (27) (28)

Tewari’s variety of solutions (2014) is given by equations (26) and (27) for various values of \( n \) and \( l \). To create a new realistic model, we take the positive constant \( c_1 = 1, d_1 + d_1 = 1 \), and assume \( l = -5 + 2\sqrt{2} \) in the context of (26) and (27) and obtain

\[
X_0 = \left( 1 + c_1 r^2 \right)^{-\frac{\sqrt{2}}{4}}
\]

\[
Y_0 = \left( 1 + c_1 r^2 \right)^{-1-\frac{\sqrt{2}}{4}}
\]

(29) (30)

The expression for pressure, density and heat flux can be obtained as

\[
\epsilon = \frac{1}{t^2(1+c_1 r^2)^{\frac{\sqrt{2}}{2}} \left[ \frac{c_1(\sqrt{2}+1)+c_1 r^2}{2a^2(1+c_1 r^2)} - 1 \right]}
\]

\[
p = \frac{1}{t^2(1+c_1 r^2)^{\frac{\sqrt{2}}{2}} \left[ \frac{c_1(-4(\sqrt{2}+1)+c_1 r^2)}{2a^2(1+c_1 r^2)} - 1 \right]}
\]

\[
q = \frac{\sqrt{2}c_1 r}{a^2 t^2(1+c_1 r^2)^{\frac{3\sqrt{2}}{4}}}
\]

(31) (32) (33)

From the above equations, we obtain

\[
\eta = -\frac{\sqrt{2}c_1 r}{2(1+c_1 r^2)^{\frac{1}{2}}}
\]

\[
\zeta = \frac{c_1(-4(\sqrt{2}+1)+c_1 r^2)}{2(1+c_1 r^2)}
\]

(34) (35)

In the solution \( f = -\alpha t \), the constant \( \alpha \) chosen to be positive \( (\eta > 0, \zeta > 0) \) so the collapsing phase corresponds to \( -\infty < t < 0 \). The isotropic pressure \( p \), density \( \epsilon \) and \( q \) have value 0 at \( t \to \infty \) and diverge when \( t \to 0 \). The matter density and isotropic pressure evolve with time as \( t^{-2} \) while \( q \) evolves as \( t^{-3} \).

4. Energy conditions

The \( \epsilon > 0, p > 0 \) are satisfied if

\[
\alpha^2 < \frac{c_1(-4(\sqrt{2}+1)+c_1 r^2)}{2(1+c_1 r^2)}
\]

(36)

and \( \epsilon > 0, p > 0 \) are satisfied if

\[
\alpha^2 < \frac{c_1(4(\sqrt{2}+1)+c_1 r^2)}{2(1+c_1 r^2)}
\]

(37)

Above conditions (36) and (37) are satisfied if \( \alpha^2 < c_1 \{-2(\sqrt{2}+1)\} \).

Now \( (\epsilon - p) > 0 \) are satisfied if

\[
\alpha^2 < \frac{-c_1(5(\sqrt{2}+8))}{4(1+c_1 r^2)}
\]

(38)
$(\epsilon - 3p) > 0$ are satisfied if

$$\alpha^2 < \frac{c_1[-3(4+3\sqrt{2})+c_1r^2]}{6(1+c_1r^2)} \quad (39)$$

Then all the physical suitable conditions will be satisfied if

$$\frac{c_1[-3(4+3\sqrt{2})+c_1r^2]}{6(1+c_1r^2)} < \alpha^2 < c_1\{-2(\sqrt{2} + 1)\}.$$  

Due to radial heat flow, the fluid must satisfy the condition $(\epsilon + p) > 2|q|$, where $|q| = (g_{ij}q^i q^j)^{\frac{1}{2}}$, to ensure consistency with all energy conditions. This would require,

$$\left[\frac{c_1(4+\sqrt{2})+c_1r^2}{2(1+c_1r^2)} + 1\right] > -\frac{\sqrt{2}c_1r}{a(1+c_1r^2)^2} \quad (40)$$

which would always be true as it could be written as

$$\left[\sqrt{2} + \frac{c_1r}{a(1+c_1r^2)^2}\right]^2 > -\frac{c_1(4+\sqrt{2})}{a^2(1+c_1r^2)} \quad (41)$$

Now we obtained the explicit formulation for surface luminosity radius, surface luminosity, total energy entrapped inside $\Sigma$ as

$$R_\Sigma(\nu) = (r')_\Sigma = -r_\Sigma at \left(1 + c_1 r^2\right)^{-\frac{2-\sqrt{2}}{4}} \quad (42)$$

$$M(\nu) = -\frac{1}{2} atr^2_\Sigma(1 + c_1 r^2)^{-\frac{6-\sqrt{2}}{4}} \left[\alpha^2 + \frac{c_1[(4-2\sqrt{2})-c_1r^2]}{2(1+c_1r^2)}\right] \quad (43)$$

In view of (3), (12), (13) and (14), we get

$$\Theta = \frac{3}{\tau(1+c_1r^2)^{\frac{1}{2}}} \quad (44)$$

$$L_\Sigma = \frac{-\sqrt{2}ac_1r^2}{(1+c_1r^2)^{\frac{3}{2}}} \quad (45)$$

$$L_\infty = \frac{-\sqrt{2}ac_1r^2}{2(1+c_1r^2)^{\frac{3}{2}}} \left[1 - \frac{\sqrt{2}}{2} c_1 r^2 - r_\Sigma \alpha (1 + c_1 r^2)\right]^{\frac{1}{2}} \quad (46)$$

$$z_\Sigma = \frac{(1+c_1r^2)}{1 - \frac{\sqrt{2}}{2} c_1 r^2 - r_\Sigma \alpha (1+c_1r^2)^{\frac{1}{2}}} - 1 \quad (47)$$

To avoid the situation of horizon appearance the model parameters might be set that $\frac{2M}{R_\Sigma} < 1$. This implies

$$\alpha^2 < \left(\frac{1-\frac{\sqrt{2}}{2} c_1 r^2}{c_1 r^2(1+c_1r^2)^{\frac{1}{2}}}\right)^2 \quad (48)$$

The analysis of temperature regarding the model is obtained from the following law given by Maartens (1995), Israel et al. (1979) and Martinez (1996).

$$\tau (g^{ij} + u^i u^j) u^a q_{\mu a} + q' = -\mathcal{K}(g^{ij} + u^i u^j)[T_j + T\dot{u}_j] \quad (48)$$

where symbols have their usual meanings.

The relaxation time in above expression (48) is given by the following relation

$$q = -\mathcal{K} \frac{1}{r^2} \left(T + T \frac{x_0}{x_0}ight) = -\frac{2x_0^2}{x_0^2/2} \quad (49)$$
The effective surface temperature given as Schwarzschild (1958) and could be evaluated with the help of following relation
\[
T^4 = \left(\frac{1}{\pi\delta(r_{\text{eq}})^2}\right) L_{\infty} = \frac{\sqrt{2} c_1 r_0^4}{2\pi\delta c_1 r_0^2 (1 + c_1 r_0^2)^{\frac{3}{2}}} \left[1 - \frac{\sqrt{2}}{2} c_1 r_0^2 - r_0^2 c_1 (1 + c_1 r_0^2)^{\frac{1}{2}}\right] \right] (50)
\]
where \( \delta = \frac{\pi^2 k^4}{15\hbar^4}\).

Choosing \( \Omega = 3 \) in Misner et al. (1964), the function \( T_0 \) is given by
\[
T_0(t) = \frac{1}{\alpha t^2} \left[\frac{8}{3\gamma(1+c_1 r_0^2)} - \frac{\sqrt{2} c_1}{\pi\delta c_1 r_0^2 (1 + c_1 r_0^2)^{\frac{3}{2}}} \right] \right] (51)
\]

The temperature inside the star is given by
\[
T^4 = \left[\frac{T_0(t)}{(1+c_1 r_0^2)^{\frac{3}{2}}} - \frac{8}{3\gamma(1+c_1 r_0^2)} \right] \right] (52)
\]

5. Graphical representations of model parameters
We have taken model parameters as following for pictorial representation
\( c_1 = 0.1 \), \( r_0 = 0.5 \).

Fig. 1 Evolution of Energy density versus radial and temporal coordinates

Fig. 2 Evolution of pressure versus radial and temporal coordinates

Fig. 3 Evolution of Heat flux versus radial and temporal coordinates

Fig. 4 Evolution of Collapse rate versus radial and temporal coordinates
6. Conclusion
The exact model of shear-free spherically symmetric isotropic fluid distributions of matter without the appearance of horizon are presented in this study. By assuming $n = -1 + \sqrt{2}$, we create a new horizon-free model. When time approaches zero, the surface temperature is zero, yet it progressively rises and eventually diverges. The luminance for a distant observer has been shown as time independent in this model. The model fulfilled all the basic properties of a collapsing radiating star. The rate
of collapse grows from the beginning condition in a realistic model proposed by Tewari et al. (2015a) and the mass-to-radius ratio is time independent. Finally, the rate of mass loss was balanced by the collapse of boundary radius; as a result, our model did not form in the horizon, and it was observed that the boundary surface could never reach the horizon when
\[ \alpha^2 < \left( 1 - \frac{\sqrt{2}}{2} c_1 r_s^2 \right)^2 / \rho_s^2 \left( 1 + c_1 r_s^2 \right). \]

References


