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A New Generalized Estimator of Population Mean Utilizing Auxiliary Parameters

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Abstract: With minimal increase in survey costs, the goal of this study was to find the population mean of the main variable. We suggest a new estimator for the population mean of the main variable in this paper, that takes advantage of readily available data on the population median of the primary variable. We investigate the bias and mean squared errors (MSE) of the introduced family of estimators up to an order one approximation. For the optimal values of the characterizing scalars, the introduced estimator has the lowest MSE. The newly presented class is compared with competing estimators that employ well-known auxiliary parameters acquired at an additional survey cost. The efficiency conditions are theoretically determined and empirically validated. Finally, the results are displayed in a tabular format. For computation work, R programming language codes were used.

Keywords: study variable • median • bias • mean square error • efficiency

Introduction

Sampling theory plays a prominent role in the estimation of unknown population parameters whenever the population is too large to handle or we have limited resources such as duration, cost and labor available for conducting the complete enumeration. It helps in inferring the population characteristics from a subset of population often known as the sample, with great precision. As the sample mean value is the best unbiased estimator of the population mean, we may infer that a parameter is best estimated by the corresponding statistic, but many times it has a significantly large sampling variance or mean square error. Thus, our focus is to obtain an estimator that probably be biased but should have minimum mean square error. Auxiliary variables fulfil this purpose as it has been well established that the correct utilization of auxiliary information brings out an increase in the precision of population parameters. Here by auxiliary information, we mean the additional or supplementary information that is not directly available in the original population but is correlated with the study variable and may be primarily utilized at sample selection as well as at estimation stage. Ratio and regression estimators are often preferred over the usually used mean estimator when the estimation procedure is assisted by auxiliary information. In general ratio estimator is more efficient, provided there is a positive correlation between the main variable(y) and auxiliary variable(x) and line of regression of y on x is linear and goes through the origin (Cochran 1940, 1942). While regression estimator is preferred when the regression line does not pass through the origin (Robson 1957; Murthy 1964). This supplementary information regarding the auxiliary variable is usually collected at an extra cost of the survey. Thus, it is suggested to suitably utilize every bit of information available about the study variable (Searls 1964, Singh and Tailor 2003) as it is quite an economical process than utilizing information about the auxiliary variables. Here in this study, the readily available information of the population median of the study variable has been utilized to estimate the mean of the main variable without expanding the expense of the survey and leading to a better estimation.

Let's consider a countable population having N unique and recognizable units. Additionally, let Y_i represent the study variable and X_i represent the auxiliary variable taking qualities $\{Y_1, Y_2, ...Y_N\}$ and $\{X_1, X_2, ... X_N\}$ correspondingly. Consider choosing a sample (x_i, y_i) of size n from (X_i, Y_i) utilizing SRSWOR approach. Let $\bar{X}$, $\bar{Y}$ represent the population means of the auxiliary and the study variable and $\bar{x}$, $\bar{y}$ be the associated sample means. It is a well-rooted theory that implies, in simple random sampling scheme sample means $\bar{x}$ and $\bar{y}$ are unbiased estimators of the population's averages $\bar{X}$ and $\bar{Y}$ respectively. And as we know that population mean is an important and extensively used measure of descriptive statistics in almost every field of science and social science including farming, clinical sciences, biological sciences, industry, business, environmental sciences and humanities and so forth. Accordingly, the estimation of population mean is of utmost importance in the aforesaid fields. In this study, we instituted a new generalized estimator of $\bar{Y}$ having higher precision under SRSWOR technique.

There have been many estimators existing in the literary text for the estimation of population mean, proposed by various researchers. In 1940 Cochran developed the typical ratio estimator of $\bar{Y}$and moving further Searls (1964) utilized coefficient of variation in estimation strategy for upgrading the effectiveness of the ratio estimator. Subramani (2013) developed a median based modified ratio estimator, Hasan (2019) developed a modified product estimator utilizing median, Yadav (2021) proposed estimator of $\bar{Y}$using known parameters of auxiliary and study variable, Sharma (2022) developed two efficient class of ratio estimator of $\bar{Y}$and so on. For more detailed study one may go through the work of Audu(2020), Baghel (2020), Bhushan (2023), Cochran (1940), Dubey (2021), Eyo (2022), Hafeez (2020), Hasan (2019), Hussain (2023) (a), (2023) (b), Muili (2019), Raja (2023), Rao (1991), Singh (2023) Subramani (2013) (a), (b), Subramani (2016), Upadhyaya (1999), Watson (1937), Yadav et al.(2023),Yadav el al.(2022), Yadav (2021a, 2021 b) and Zaman (2019).

Some of these estimators, with their bias and mean square error terms are listed in this section for the cause of comparing their efficiency with the proposed estimator t_p in the upcoming section.

Review of Literature

1. The most usual and unbiased estimator $\bar{y}$ of $\bar{Y}$denoted and defined as,

$$t_o = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad (1)$$

With sampling variance given by,

$$Var(t_0) = \frac{1-f}{n}S_y^2 = \frac{1-f}{n}\bar{Y}^2 C_y^2 = \lambda \bar{Y}^2 C_y^2 \qquad (2)$$

2. When supported by the additional information, Cochran (1940) has developed more efficient ratio estimator denoted and defined as,

$$t_1 = \frac{\bar{y}}{\bar{x}}\bar{X} \qquad (3)$$

Bias(t_1) $= \frac{1-f}{n}\bar{Y}[C_x^2 - C_{yx}] = \lambda\bar{Y}[C_x^2 - C_{yx}]$

(4)

MSE(t_1) $= \frac{1-f}{n}\bar{Y}^2[C_y^2 + C_x^2 - 2C_{yx}] = \lambda\bar{Y}^2[C_y^2 + C_x^2 - 2C_{yx}]$

(5)

3. Subramani et al. (2013), defined a modified ratio type estimator by putting to use median (M) of the auxiliary variable defined as,

$t_2 = \bar{y}\left[\frac{\bar{X}+M}{\bar{x}+M}\right]$

(6)

Bias(t_2) $= \frac{1-f}{n}\bar{Y}\left[\left(\frac{\bar{X}}{\bar{X}+M}\right)^2 C_x^2 - 2\left(\frac{\bar{X}}{\bar{X}+M}\right)\rho C_y C_x\right] = \lambda\bar{Y}[R_3^2 C_x^2 - 2R_3\rho C_y C_x]$

(7)

MSE(t_2) $= \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + \left(\frac{\bar{X}}{\bar{X}+M}\right)^2 C_x^2 - 2\left(\frac{\bar{X}}{\bar{X}+M}\right)\rho C_y C_x\right] = \lambda\bar{Y}^2[C_y^2 + R_3^2 C_x^2 - 2R_3\rho C_y C_x]$

(8)

4. Watson (1937) developed an unbiased linear regression estimator defined as,

$t_3 = \bar{y} + b(\bar{X} - \bar{x})$

(9)

With sampling variance given by,

Var(t_3) $= \frac{1-f}{n}\bar{Y}^2 C_y^2[1 - \rho_{yx}^2] = \lambda\bar{Y}^2 C_y^2[1 - \rho_{yx}^2]$

(10)

5. Kadilar (2016), worked out a new exponential ratio estimator denoted by,

$t_4 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\delta} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$

(11)

Bias(t_4) $= \frac{1-f}{n}\bar{Y}\left[\left(\frac{\delta(\delta-1)}{2} + \frac{3}{8}\right)C_x^2 + \left(\delta + \frac{1}{2}\right)C_{yx}\right]$

(12)

MSE(t_4) $= \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + \left(\delta^2 + \delta + \frac{1}{4}\right)C_x^2 + (2\delta + 1)C_{yx}\right]$

(13)

Optimum value of δ for minimizing the value of MSE is,

$\delta_{opt} = \left[\frac{1}{2} - \rho_{yx}\frac{C_y}{C_x}\right]$

Hence minimum MSE of estimator t_4 is obtain as,

MSE(t_4) $= \frac{1-f}{n}\bar{Y}^2 C_y^2(1 - \rho_{yx}^2) = \lambda\bar{Y}^2 C_y^2(1 - \rho_{yx}^2)$

(14)

6. Subramani (2016) in order to estimate $\bar{Y}$, developed a population median based, ratio estimator given by,

$t_5 = \bar{y}\left[\frac{M}{m}\right]$

(15)

Bias(t_5) = $\frac{1-f}{n}\overline{Y}\left[C_m^2 - C_{ym} - \frac{Bias(m)}{M}\right] = \lambda\overline{Y}\left[C_m^2 - C_{ym} - \frac{Bias(m)}{M}\right]$
(16)

MSE(t_5) = $\frac{1-f}{n}\overline{Y}^2\left[C_y^2 + \left(\frac{\overline{Y}}{M}\right)^2 C_m^2 - 2\frac{\overline{Y}}{M}C_{ym}\right] = \lambda\overline{Y}^2\left[C_y^2 + R_5^2C_m^2 - 2R_5C_{ym}\right]$
(17)

where, various notations are defined as,

$R_5 = \frac{\overline{Y}}{M}$, $C_m = \frac{S_m}{M}$, $C_{ym} = \frac{S_{ym}}{\overline{Y}M}$, $S_m^2 = \frac{1}{{}^NC_n}\sum_{i=1}^{{}^NC_n}(m_i - M)^2$, $S_{ym} = \frac{1}{{}^NC_n}\sum_{i=1}^{{}^NC_n}(\overline{y}_i - \overline{Y})(m_i - M)$

Proposed Estimator

Taking inspiration from the work of Kadilar (2016), Subramani (2016), we proposed or instituted a new generalized estimator of $\overline{Y}$ by putting to use the known information about the median of the study variable, it is denoted and defined as:

$t_p = \overline{y}\left(\frac{M}{m}\right)^{\alpha}\left(1 + \log\frac{m}{M}\right)^{\beta}$
(18)

where α and β are characterizing scalars whose value to be found in such a way that the MSE of the instituted estimator is minimum.

Consider the accompanying approximations for finding the equation of bias and MSE of instituted estimator t_p .

$\overline{y} = \overline{Y}$ (1 + e_0) and m = M (1 + e_1), in such a manner that

E(e_0) = 0, E(e_1) = $\frac{\overline{M}-M}{M} = \frac{Bias(m)}{M}$, E($e_0^2$) = $\frac{1-f}{n}C_y^2 = \lambda C_y^2$, E($e_1^2$) = $\frac{1-f}{n}C_m^2 = \lambda C_m^2$ E(e_0e_1) = $\frac{1-f}{n}$ C_{ym}

and $\overline{M} = \frac{1}{n}\sum_{i=1}^{n}m_i$

Putting the above-mentioned value of $\overline{y}$ and 'm' in expression (2), t_p can be re-written as

$t_p = \overline{Y}(1 + e_0)\left(\frac{M}{M(1+e_1)}\right)^{\alpha}\left(1 + \log\frac{M(1+e_1)}{M}\right)^{\beta}$

$= \overline{Y}(1 + e_0)(1 + e_1)^{-\alpha}(1 + \log(1 + e_1))^{\beta}$

$= \overline{Y}(1 + e_0)\left[1 - \alpha e_1 + \frac{\alpha(\alpha+1)e_1^2}{2} + \dots\right]\left(1 + e_1 - e_1^2 + \dots\right)^{\beta}$

expanding RHS of the above expression up to first level of approximation and discarding the particulars of e's containing order higher than two, we obtain

$t_p = \overline{Y}\left[1 + e_0 - \alpha e_1 + \alpha e_0e_1 + \frac{\alpha(\alpha+1)e_1^2}{2} + \beta e_1 + \beta e_0e_1 - \alpha\beta e_1^2 + (\beta_2 - 3\beta)\frac{e_1^2}{2}\right]$
(19)

Subtracting $\overline{Y}$ from both side of (19) and afterward taking expectation and rearranging the terms we acquire the condition of bias of estimator t_p as follows

E (t_p - $\overline{Y}$) = Bias(t_p) = $\overline{Y}E\left[e_0 + (\beta - \alpha)e_1 + (\beta - \alpha)e_0e_1 + \left(\frac{\alpha(\alpha+1)}{2} - \alpha\beta + \frac{\beta(\beta-3)}{2}\right)e_1^2\right]$
(20)

Putting the values of different expectation in (20), we obtain bias as

Bias(t_p) = $\overline{Y}\left[(\beta-\alpha)\frac{Bias(m)}{M}+(\beta-\alpha)\lambda C_{ym}^2+\left(\frac{\alpha(\alpha+1)}{2}-\alpha\beta+\frac{\beta(\beta-3)}{2}\right)\lambda C_m^2\right]$ (21)

Again, subtracting $\overline{Y}$ from both side of (19), squaring and then taking expectation and rearranging the terms we get the equation of MSE of estimator t_p as follows

$$E(t_p-\overline{Y})^2 = MSE(t_p) = \overline{Y}^2 E[e_0^2+(\beta-\alpha)^2 e_1^2+2(\beta-\alpha)e_0e_1]$$

Putting values of different expectations and (β - α) = γ we get

$$MSE(t_p) = \overline{Y}^2[\lambda C_Y^2+\gamma^2\lambda C_m^2+2\gamma\lambda C_{ym}^2] \quad (22)$$

Value of γ for which MSE of t_p is minimum is given by

$$\gamma = -\frac{C_{ym}}{C_m^2}, \quad (23)$$

Putting in equation (6), minimum MSE of estimator t_p is

$$MSE_{min} = \frac{1-f}{n}\overline{Y^2}\left[C_y^2-\frac{C_{ym}^2}{C_m^2}\right] = \lambda\overline{Y^2}\left[C_y^2-\frac{C_{ym}^2}{C_m^2}\right] \quad (24)$$

Efficiency Comparison Study

Theoretical Comparison

The effectiveness of the instituted estimator t_p has been checked over the other competing estimators $t_i(i=0,1,2,3,4,5)$ in the following section that are already presented in literary works. This comparison provided the constraints that the instituted estimator t_p, operates under to estimate $\overline{Y}$ more efficiently than the competing estimators.

If $MSE(t_p) < MSE(t_i)$/var(t_i), where i = (1,2,3,4,5), we obtained the conditions under which t_p is efficient than $t_i's$ as follows

1.Equations (24) and (2) implies that the instituted estimator t_p is more efficient than t_0 if, $\frac{C_{ym}^2}{C_m^2}<0$ i.e., $C_{ym}^2<0$ (25)

2.Equations (24) and (5) implies that t_p is more efficient than t_1 if, $2C_{yx}<C_x^2+\frac{C_{ym}^2}{C_m^2}$ (26)

3.From equations (24) and (8) we get, t_p is more efficient than the existing estimator t_2 if, $2R_3C_{yx}<R_3^2C_x^2+\frac{C_{ym}^2}{C_m^2}$ (27)

4.From equations (24) and (10) we conclude that t_p is prefer over t_3 if, $\frac{C_{ym}^2}{C_m^2}-\rho_{yx}^2C_y^2>0$ (28)

5. Utilizing equations (24) and (13) we obtain t_p is efficient than t_4 if, $\frac{C_{ym}^2}{C_m^2}-\rho_{yx}^2C_y^2>0$ (29)

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6. And from equations (24) and (17) we observe that, t_p is more efficient than the existing estimator t_5 if,

$$2R_5C_{ym} < R_5^2C_m^2 + \frac{C_{ym}^2}{C_m^2} \quad (30)$$

Numerical Computation and Comparison

Here, we evaluated the effectiveness of the instituted estimator t_p by taking into consideration the three real population data sets illustrated as below:

Population description

Population I

Y- Area of cultivation for wheat crop in 1974, X- Area of cultivation for wheat crop in 1971.

Population II

Y- Area of cultivation for wheat crop in 1974, X- Area of cultivation for wheat crop in 1973.

Population III

Y- Raw material quantity (in lakhs of bales) of 20 jute mills, X- Number of laborers' (in thousand).

Table 1. Parameters and constants values enumerated from the mentioned populations**.**

Parameters	**Population I**	**Population II**	**Population III**
N	34	34	20
n	3	3	3
${}^{N}C_n$	5984	5984	1140
$\overline{Y}$	856.4118	856.4118	41.5
$\overline{M}$	747.7223	747.7223	40.2351
M	767.5	767.5	40.5
$\overline{X}$	208.8824	199.4412	441.95
R_1	4.0999	4.2941	0.0939
R_3	0.2139	0.2062	0.3654
R_5	1.1158	1.1158	1.0247
C_y^2	0.222726	0.222726	0.01575
C_x^2	0.157785	0.172408	0.014818
C_m^2	0.172341	0.172341	0.015931
C_{ym}^2	0.137284	0.137284	0.012549
C_{yx}^2	0.084194	0.087264	0.009964
ρ_{yx}	0.4491	0.4453	0.6522

Source Population I, II, III [Subramani and Prabhavathy]

Table 2. Value of Bias of the proposed and the competing estimators for aforesaid populations.

Estimator	**Population I**	**Population II**	**Population III**
t_1	19.154439	22.16145	0.057075
t_2	-7.494025	-7.460161	-0.068431
t_4	26.33463	28.18379	0.1213281
t_5	16.00932	16.00932	0.1171812

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Table 3. Values of MSE\Variance of the proposed and the competing estimators for aforesaid populations.

Estimator	Population I	Population II	Population III
t_o	49647.54	49647.54	7.69
t_1	47284.04	49174.97	5.19
t_2	49142.27	49550.40	5.40
t_3	39634.09	39802.82	4.42
t_4	39634.09	39802.82	4.42
t_5	29185.25	29185.25	3.29
t_p	25270.66	25270.66	2.86

In the bar graphs from figure 1 to 3, we have visually displayed the values of MSE/Variance of the instituted and the competing estimators for the three population data sets having sample size n = 3.

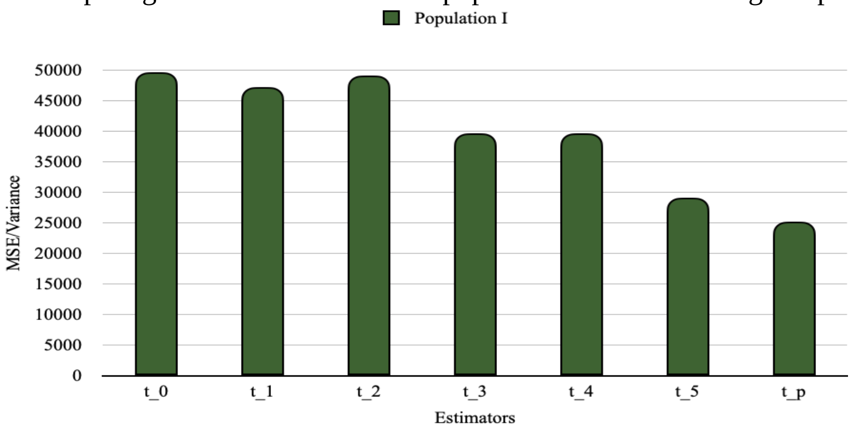

Fig 1. Displays the MSE/Variance values of estimators for dataset of Population I

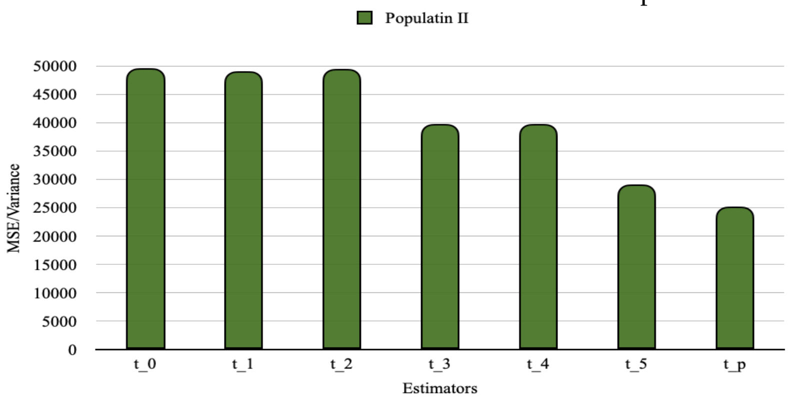

Fig 2. Show the MSE/Variance outcomes of estimators for dataset of Population II

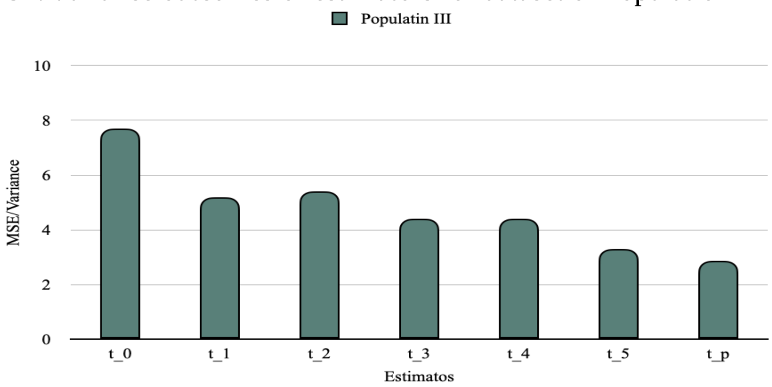

Fig 3. Displays the MSE/Variance outcomes of estimators for dataset of Population III

Conclusion

In this proposed work, we have instituted a new generalized estimator t_p for the estimation of population mean $\bar{Y}$, by utilizing known statistics about the median of the study variable under SRSWOR. Then we established the expressions of bias and MSE of the instituted estimator up to an order one of approximation and then compared theoretically with the MSE/Variance of the competing modified estimators namely t_o, t_1, t_2, t_3, t_4, and t_5 leading to inferred the constraints under which t_p is more efficient than the competing estimators and furthermore, the proficiency of the instituted estimator have been evaluated with the help of a numerical example. t_p is highly advocated to be used in the field of clinical sciences, business management, economics, agriculture sciences and humanities etc. as it provides a more refined estimate of population mean without expanding the expenses of the survey.

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