



## A Common Fixed Point Theorem For Generalized $(\psi, \phi)$ Contraction In Complete B-Metric Spaces

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**Abstract:** In this paper, we obtain a common fixed point result for weakly compatible mapping in b-metric like spaces. We generalize and extend various fixed point theorems of the literature. In addition, we attempt to make some important conclusions for the established result.

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**Keywords:** coincidence point • weakly compatible • common fixed point • b-metric like space

### Introduction and Preliminaries

Mathews (1994) introduced a generalization of the metric space, popularly known as partial metric space, in which self-distance may be non-zero. Harandi (2012) further generalized this notion by introducing a new space named the metric-like-space in which the self-distance of a point may be greater than the distance of that point to any other point. Meanwhile, Alghamdi et al. (2013) introduced the concept of b-metric-like spaces that generalized the notions of partial b-metric space and metric-like space. Afterwards, Shukla (2014), also introduced an idea of the partial b-metric which generalizes of the partial metric and b-metric, altogether.

Zoto et. al. (2017) established some fixed point theorems in generalized  $(\alpha - \psi, \phi)$  contractions in b- metric-like spaces. Also, Guan and Li (2021) proved some common fixed point results for generalized  $(\psi, \phi)$  weakly contractive mappings in b-metric like spaces. Besides, many authors developed various fixed point results in metric-like spaces and generalized metric-like spaces (Aydi et al. 2017, Chen et al. 2015, Guan et al.2021 , Hammad et al. 2019, Harandi 2012, Hussain et al. 2014, Sen et al. 2019 , Shah 2022), Sumalai et al.2019), Zoto et al. 2018,2019).

We define the following class of functions.

1.  $\Psi = \{\psi: [0, +\infty) \rightarrow [0, +\infty) \text{ is continuous and non-decreasing with } \psi(t) = 0 \text{ if and only if } t = 0\}$
2.  $\Phi = \{\phi: [0, +\infty) \rightarrow [0, +\infty) \text{ is lower semi continuous with } \phi(t) > 0 \text{ for all } t > 0 \text{ and } \phi(0) = 0\}$ .



Now we recall some important definitions and other consequences from the availed literature. The notion of metric-like space due to A. Harandi is defined as follows:

**Definition 1.1 (Harandi (2012))** Let  $X$  be a non-empty set. Then, a mapping  $d: X \times X \rightarrow [0, \infty)$  is said to be a metric-like if for all  $x, y, z \in X$ ,

1.  $d(x, y) = 0 \Rightarrow x = y$ ;
2.  $d(x, y) = d(y, x)$ ;
3.  $d(x, z) \leq d(x, y) + d(y, z)$ .

Then, the pair  $(X, d)$  is known as metric-like space.

Alghamdi et al. (2013) defined the concept of b-metric like space as follows:

**Definition 1.2 (Alghamdi et al., 2013)** Let  $X$  be a non-empty set. Then, a mapping  $d: X \times X \rightarrow [0, \infty)$  is said to be a  $b$ -metric-like if there exists a number  $s \geq 1$  such that for all  $x, y, z \in X$ ,

1.  $d(x, y) = 0 \Rightarrow x = y$ ;
2.  $d(x, y) = d(y, x)$ ;
3.  $d(x, z) \leq s[d(x, y) + d(y, z)]$ .

Then, the pair  $(X, d)$  is known as  $b$ -metric-like space.

**Definition 1.3 (Alghamdi et al., 2013)** Let  $(X, d)$  be a  $b$ -metric like space with parameter  $s \geq 1$  and  $\{x_n\}$  be a sequence in  $X$ .

1. The sequence  $\{x_n\}$  is said to be convergent to  $x$  if  $\lim_{n \rightarrow +\infty} d(x_n, x) = d(x, x)$ .
2. The sequence  $\{x_n\}$  is said to be Cauchy sequence if and only if  $\lim_{n \rightarrow +\infty} d(x_n, x_m)$

exists and is finite.

3.  $(X, d)$  is said to be complete iff for every Cauchy sequence  $\{x_n\}$  in  $X$ , there exists an  $x \in X$  such that  $\lim_{n, m \rightarrow +\infty} d(x_n, x_m) = \lim_{n \rightarrow +\infty} d(x_n, x) = d(x, x)$ .

**Lemma 1.1 (Sen et al., 2019)** Let  $(X, d, s \geq 1)$  be a  $b$ -metric-like spaces and  $\{x_n\}$  be a sequence in  $X$  such that

$$d(x_n, x_{n+1}) \leq \lambda d(x_{n-1}, x_n),$$

for some  $\lambda \in [0, 1)$  and for each  $n \in \mathbb{N}$ . Then  $\{x_n\}$  is a Cauchy sequence with

$$\lim_{n, m \rightarrow +\infty} d(x_n, x_m) = 0.$$



**Lemma 1.2 (Alghamdi et al., 2013)** Let  $(X, d)$  be a  $b$ -metric like space with  $s \geq 1$ . We assume that  $\{x_n\}$  and  $\{y_n\}$  are convergent to  $x$  and  $y$ , respectively. Then, we have

$$\begin{aligned} \frac{1}{s^2}d(x, y) - \frac{1}{s}d(x, x) - d(y, y) &\leq \liminf_{n \rightarrow +\infty} d(x_n, y_n) \\ &\leq \limsup_{n \rightarrow +\infty} d(x_n, y_n) \\ &\leq sd(x, x) + s^2d(y, y) + s^2d(x, y). \end{aligned}$$

In particular, if  $d(x, y) = 0$ , then we have  $\lim_{n \rightarrow +\infty} d(x_n, y_n) = 0$ . Moreover, for each  $z \in X$ , we have

$$\frac{1}{s}d(x, z) - d(x, x) \leq \liminf_{n \rightarrow +\infty} d(x_n, z) \leq \limsup_{n \rightarrow +\infty} d(x_n, z) \leq sd(x, z) + sd(x, x).$$

Also, if  $d(x, x) = 0$  then

$$\frac{1}{s}d(x, z) \leq \liminf_{n \rightarrow +\infty} d(x_n, z) \leq \limsup_{n \rightarrow +\infty} d(x_n, z) \leq sd(x, z).$$

**Lemma 1.3 (Zoto et al., 2017)** Let  $(X, d)$  be a  $b$ -metric like space with  $s \geq 1$ . Then,

1. If  $d(x, y) = 0$  then  $d(x, x) = d(y, y) = 0$ .
2. If  $\{x_n\}$  is a sequence such that  $\lim_{n \rightarrow +\infty} d(x_n, x_{n+1}) = 0$ , then we have  $\lim_{n \rightarrow +\infty} d(x_n, x_n) = \lim_{n \rightarrow +\infty} d(x_{n+1}, x_{n+1}) = 0$
3. If  $x \neq y$  then  $d(x, y) > 0$ .

In the next section, we state and prove a common fixed point result for generalized  $(\psi, \phi)$  weak contraction in a complete  $b$ -metric like space by using the notion of weakly compatible.

## Main Results

**Theorem 2.1** Let  $(X, d, s)$  be a complete  $b$ -metric like space and let  $S, T: X \rightarrow X$  be two self mappings satisfying  $T(X) \subseteq S(X)$ , where  $S(X)$  is a closed subset of  $X$ . If there are functions  $\psi \in \Psi$  and  $\phi \in \Phi$  such that for all  $x, y \in X$ ,

$$\psi(s^2d(Tx, Ty)) \leq \psi(M_1(x, y)) - \phi(N_1(x, y)), \tag{2.1}$$

where

$$M_1(x, y) = \max \left\{ d(Ty, Sy) \left( \frac{1+d(Sx, Tx)}{1+d(Sx, Sy)} \right), d(Tx, Sx), d(Sx, Sy), \frac{1}{2s} [d(Ty, Sy) + d(Tx, Sx)], d(Tx, Sx) \left( \frac{1+d(Sx, Sy)}{1+d(Ty, Sy)} \right) \right\}$$

and



$$N_1(x, y) = \max\{d(Ty, Sy), d(Tx, Ty), d(Tx, Sy), d(Sx, Sy)\}.$$

Then  $S$  and  $T$  have a unique coincidence point in  $X$ . Moreover,  $S$  and  $T$  have a unique common fixed point provided that  $S$  and  $T$  are weakly compatible.

*Proof.* Let  $x_0 \in X$  be an arbitrary. As  $T(X) \subseteq S(X)$ , there exists  $x_1 \in X$  such that  $Tx_0 = Sx_1$ . Now, we define sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  by  $y_n = Tx_n = Sx_{n+1}$ , for all  $n \in \mathbb{N}$ . If  $y_n = y_{n+1}$  for some  $n \in \mathbb{N}$ , then we have  $y_n = y_{n+1} = Tx_{n+1} = Sx_{n+1}$  and  $T$  and  $S$  have a coincidence point.

Without loss of generality we assume that  $y_n \neq y_{n+1}$ . By Lemma 1.3, we have  $d(y_n, y_{n+1}) > 0$  for all  $n \in \mathbb{N}$ . Apply 2.1 with  $x = x_n$  and  $y = x_{n+1}$ , we obtain

$$\begin{aligned} \psi(s^2 d(y_n, y_{n+1})) &= \psi(s^2 d(Tx_n, Tx_{n+1})) \\ &\leq \psi(M_1(x_n, x_{n+1})) - \phi(N_1(x_n, x_{n+1})). \end{aligned}$$

Here,

$$M_1(x_n, x_{n+1}) = \max \left\{ \begin{array}{l} d(y_{n+1}, y_n) \left( \frac{(1+d(y_{n-1}, y_n))}{1+d(y_n, y_{n+1})} \right), \\ d(y_n, y_{n-1}), d(y_{n-1}, y_n), \\ \frac{1}{2s} [d(y_n, y_{n+1}) + d(y_n, y_{n-1})], \\ d(y_{n+1}, y_n) \left( \frac{(1+d(y_n, y_{n-1}))}{1+d(y_{n+1}, y_n)} \right) \end{array} \right\},$$

and

$$N_1(x_n, x_{n+1}) = \max\{d(y_{n+1}, y_n), d(y_n, y_{n+1}), d(y_n, y_n), d(y_{n-1}, y_n)\}.$$

If  $d(y_n, y_{n+1}) \geq d(y_n, y_{n-1}) > 0$ , then  $M_1 \leq d(y_{n+1}, y_n)$  and  $N_1 \geq d(y_{n+1}, y_n)$ . Now,

$$\begin{aligned} \psi(d(y_n, y_{n+1})) &\leq \psi(s^2 d(y_n, y_{n+1})) \\ &\leq \psi(M_1(x_n, x_{n+1})) - \phi(N_1(x_n, x_{n+1})) \\ &\leq \psi(d(y_n, y_{n+1})) - \phi(d(y_{n+1}, y_n)), \end{aligned}$$

which gives that  $d(y_n, y_{n+1}) = 0$ , a contradiction. Thus  $d(y_n, y_{n+1}) > 0$ . It follows that  $d(y_n, y_{n+1}) < d(y_n, y_{n-1})$ . Hence  $\{d(y_n, y_{n+1})\}$  is a non-increasing sequence.

Consequently, the limit of the sequence is a non-negative say  $r \geq 0$ . That is

$$\lim_{n \rightarrow \infty} d(y_n, y_{n+1}) = r. \text{ Therefore}$$

$$\begin{aligned} M_1(x_n, x_{n+1}) &\leq d(y_n, y_{n-1}) \\ N_1(x_n, x_{n+1}) &\geq d(y_n, y_{n-1}), \end{aligned}$$

so,

$$\begin{aligned} \psi(d(y_n, y_{n+1})) &\leq \psi(s^2 d(y_n, y_{n+1})) \\ &\leq \psi(s^2 d(Tx_n, Tx_{n+1})) \\ &\leq \psi(M_1(x_n, x_{n+1})) - \phi(N_1(x_n, x_{n+1})) \end{aligned}$$



$$\leq \psi(d(y_n, y_{n-1})) - \phi(d(y_n, y_{n-1})).$$

If  $r > 0$ , then letting  $n \rightarrow \infty$  in above inequalities, we obtain that  $\psi(r) \leq \psi(r) - \phi(r) \Rightarrow r = 0$ . That is  $\lim_{n \rightarrow \infty} d(y_n, y_{n+1}) = 0$ .

Now we prove that  $\lim_{n,m \rightarrow \infty} d(y_n, y_m) = 0$ . If not, there exists  $\epsilon > 0$  for which, we can find sequences  $\{y_{m_k}\}$  and  $\{y_{n_k}\}$  of  $\{y_n\}$ , so that  $n_k$  is the smallest index for which  $n_k > m_k > k$ ,  $\epsilon \leq d(y_{m_k}, y_{n_k})$  and  $d(y_{m_k}, y_{n_k-1}) < \epsilon$ . In view of triangle inequalities in b-metric like space the following inequalities hold:

$$\begin{aligned} \epsilon &\leq \limsup_{k \rightarrow \infty} d(y_{m_k}, y_{n_k}) \leq s\epsilon, \\ \frac{\epsilon}{s} &\leq \limsup_{k \rightarrow \infty} d(y_{m_k}, y_{n_k-1}) \leq \epsilon, \\ \frac{\epsilon}{s} &\leq \limsup_{k \rightarrow \infty} d(y_{m_k-1}, y_{n_k}) \leq s^2\epsilon, \\ \frac{\epsilon}{s^2} &\leq \limsup_{k \rightarrow \infty} d(y_{m_k-1}, y_{n_k-1}) \leq s\epsilon. \end{aligned}$$

Also, we find

$$\begin{aligned} M_1(x_{m_k}, x_{n_k}) &= \max \left\{ \begin{aligned} &d(Tx_{n_k}, Sx_{n_k}) \left( \frac{1+d(Sx_{m_k}, Tx_{m_k})}{1+d(Sx_{m_k}, Sx_{n_k})} \right), \\ &d(Tx_{m_k}, Sx_{m_k}), d(Sx_{m_k}, Sx_{n_k}) \\ &\frac{1}{2s} [d(Tx_{n_k}, Sx_{n_k}) + d(Tx_{m_k}, Sx_{m_k})], \\ &d(Tx_{m_k}, Sx_{m_k}) \left( \frac{1+d(Sx_{m_k}, Sx_{n_k})}{1+d(Tx_{n_k}, Sx_{n_k})} \right) \end{aligned} \right\} \\ &= \max \left\{ \begin{aligned} &d(y_{n_k}, y_{n_k-1}) \left( \frac{1+d(y_{m_k-1}, y_{m_k})}{1+d(y_{m_k-1}, y_{n_k-1})} \right), \\ &d(y_{m_k}, y_{m_k-1}), d(y_{m_k-1}, y_{n_k-1}), \\ &\frac{1}{2s} [d(y_{n_k}, y_{n_k-1}) + d(y_{m_k}, y_{m_k-1})], \\ &d(y_{m_k}, y_{m_k-1}) \left( \frac{1+d(y_{m_k-1}, y_{n_k-1})}{1+d(y_{n_k}, y_{n_k-1})} \right) \end{aligned} \right\}, \end{aligned}$$

and

$$\begin{aligned} N_1(x_{m_k}, x_{n_k}) &= \max \left\{ \begin{aligned} &d(Tx_{n_k}, Sx_{n_k}), d(Tx_{m_k}, Tx_{n_k}), \\ &d(Tx_{m_k}, Sx_{n_k}), d(Sx_{m_k}, Sx_{n_k}) \end{aligned} \right\} \\ &= \max \left\{ \begin{aligned} &d(y_{n_k}, y_{n_k-1}), d(y_{m_k}, y_{n_k}), \\ &d(y_{m_k}, y_{n_k-1}), d(y_{m_k-1}, y_{n_k-1}) \end{aligned} \right\}. \end{aligned}$$

Thus, using above four inequalities, we have

$$\limsup_{k \rightarrow \infty} M_1(x_{m_k}, x_{n_k}) \leq \max\{0, 0, s\epsilon, 0, 0\} \leq s\epsilon.$$

$$\liminf_{k \rightarrow \infty} N_1(x_{m_k}, x_{n_k}) \geq \max\left\{0, \epsilon, \frac{\epsilon}{s}, \frac{\epsilon}{s^2}\right\} \geq \frac{\epsilon}{s^2}.$$

Taking  $x = x_{m_k}$  and  $y = x_{n_k}$  in equation 2.1, we get



$$\begin{aligned} \psi(d(y_{m_k}, y_{n_k})) &\leq \psi(s^2 d(y_{m_k}, y_{n_k})) \\ &\leq \psi(s^2 d(Tx_{m_k}, Tx_{n_k})) \\ &\leq \psi(M_1(x_{m_k}, x_{n_k})) - \phi(N_1(x_{m_k}, x_{n_k})) \\ &\leq \psi(d(y_n, y_{n-1})) - \phi(d(y_n, y_{n-1})). \end{aligned}$$

So, we find

$$\begin{aligned} \psi(s\epsilon) &\leq \psi(s \limsup_{k \rightarrow \infty} d(Tx_{m_k}, Tx_{n_k})) \\ &\leq \psi(s^2 \limsup_{k \rightarrow \infty} d(Tx_{m_k}, Tx_{n_k})) \\ &\leq \psi(\limsup_{k \rightarrow \infty} M_1(x_{m_k}, x_{n_k})) - \phi(\liminf_{k \rightarrow \infty} N_1(x_{m_k}, x_{n_k})) \\ &\leq \psi(s\epsilon) - \phi(\liminf_{k \rightarrow \infty} N_1(x_{m_k}, x_{n_k})), \end{aligned}$$

and we conclude that  $\lim_{k \rightarrow \infty} \inf N_1(x_{m_k}, x_{n_k}) = 0$ , otherwise it gives a contradiction to  $\lim_{k \rightarrow \infty} \inf N_1(x_{m_k}, y_{n_k}) \geq \frac{\epsilon}{s^2}$ . Hence  $\lim_{n,m \rightarrow \infty} d(y_n, y_m) = 0$ .

As  $X$  is complete so there exists  $u \in X$ , such that

$$\lim_{n \rightarrow \infty} d(y_n, u) = \lim_{n \rightarrow \infty} d(Tx_n, u) = \lim_{n \rightarrow \infty} d(Sx_{n+1}, u) = \lim_{n,m \rightarrow \infty} d(y_n, y_m) = d(u, u) = 0.$$

Since  $S(X)$  is closed, we obtain that  $u \in S(X)$ . Therefore, one can choose  $z \in X$  such that  $u = Sz$  and above equality becomes

$$\lim_{n \rightarrow \infty} d(y_n, Sz) = \lim_{n \rightarrow \infty} d(Tx_n, Sz) = \lim_{n \rightarrow \infty} d(Sx_{n+1}, Sz) = 0.$$

If  $Tz \neq Sz$ , by putting  $x = x_n$  and  $y = z$  in equation 2.1, we get

$$\psi(s^2 d(Tx_n, Tz)) \leq \psi(M_1(x_n, z)) - \phi(N_1(x_n, z)),$$

where

$$M_1(x_n, z) = \max \left\{ \begin{aligned} &d(Tz, Sz) \left( \frac{1+d(Sx_n, Tx_n)}{1+d(Sx_n, Sz)} \right), d(Tx_n, Sx_n), \\ &d(Sx_n, Sz), \frac{1}{2s} [d(Tz, Sz) + d(Tx_n, Sx_n)], \\ &d(Tx_n, Sx_n) \left( \frac{1+d(Sx_n, Sz)}{1+d(Tz, Sz)} \right) \end{aligned} \right\}$$

and

$$N_1(x_n, z) = \max\{d(Tz, Sz), d(Tx_n, Tz), d(Tx_n, Sz), d(Sx_n, Sz)\}.$$

Consequently, we get

$$\begin{aligned} \limsup_{k \rightarrow \infty} M_1(x_n, z) &= \max \left\{ \begin{aligned} &d(Tz, Sz) \left( \frac{1+d(y_{n-1}, y_n)}{1+d(y_{n-1}, Sz)} \right), 0, \frac{1}{2s} d(Tz, Sz), \\ &d(y_n, y_{n-1}) \left( \frac{1+d(y_{n-1}, Sz)}{1+d(Tz, Sz)} \right) \end{aligned} \right\} \\ &\leq \max \left\{ \begin{aligned} &d(Tz, Sz) \left( \frac{1}{1+d(y_{n-1}, Sz)} \right), 0, \\ &\frac{1}{2s} d(Tz, Sz), 0 \end{aligned} \right\} \\ &\leq d(Tz, Sz), \end{aligned}$$

and



$$\begin{aligned} \liminf_{k \rightarrow \infty} N_1(x_n, z) &= \max\{d(Tz, Sz), d(y_n, Tz), 0, d(y_{n-1}, Sz)\} \\ &\geq \max\{d(Tz, Sz), d(y_n, Tz), d(y_{n-1}, Sz)\} \\ &\geq d(Tz, Sz). \end{aligned}$$

Taking the upper limit as  $k$  tends to  $\infty$ , we have

$$\begin{aligned} \psi(d(Sz, Tz)) &= \psi(s^2 \frac{1}{s^2} d(Sz, Tz)) \\ &\leq \psi(s^2 \limsup_{k \rightarrow \infty} d(Tx_{n_k}, Tz)) \\ &\leq \psi(\limsup_{k \rightarrow \infty} M_1(x_{n_k}, z)) - \phi(\liminf_{k \rightarrow \infty} N_1(x_{n_k}, z)) \\ &\leq \psi(d(Sz, Tz)) - \phi(d(Sz, Tz)), \end{aligned}$$

which implies  $\phi(d(Sz, Tz)) = 0$ . It follows that  $d(Sz, Tz) = 0$ . That is  $u = Sz = Tz$  is a point of coincidence of  $S$  and  $T$ .

We also conclude that the point of coincidence is unique. Let  $v$  be another coincidence point with  $z \neq v$  such that  $Sv = Tv$ . Put  $x = z$  and  $y = v$  in equation 2.1, we obtain

$$\psi(s^2 d(Tz, Tv)) \leq \psi(M_1(z, v)) - \phi(N_1(z, v)),$$

where

$$M_1(z, v) = \max \left\{ \begin{aligned} &d(Tv, Sv) \left( \frac{1+d(Sz, Tz)}{1+d(Sz, Sv)} \right), d(Tz, Sz), d(Sz, Sv), \\ &\frac{1}{2s} [d(Tv, Sv) + d(Tz, Sz)], d(Tz, Sz) \left( \frac{1+d(Sz, Sv)}{1+d(Tv, Sv)} \right) \end{aligned} \right\}$$

and

$$N_1(z, v) = \max\{d(Tv, Sv), d(Tz, Tv), d(Tz, Sv), d(Sz, Sv)\}.$$

Consequently, we get

$$\begin{aligned} \limsup_{k \rightarrow \infty} M_1(z, v) &= \max \left\{ \begin{aligned} &d(Tv, Sv) \left( \frac{1}{1+d(Sz, Sv)} \right), 0, \\ &d(Sz, Sv), \frac{1}{2s} d(Tv, Sv), 0 \end{aligned} \right\} \\ &\leq \max \left\{ \begin{aligned} &d(Tv, Sv) \left( \frac{1}{1+d(Sz, Sv)} \right), \\ &d(Tz, Tv), \frac{1}{2s} d(Tv, Sv) \end{aligned} \right\} \\ &\leq d(Tz, Tv), \end{aligned}$$

and

$$\begin{aligned} \liminf_{k \rightarrow \infty} N_1(z, v) &= \max\{d(Tv, Sv), d(Tz, Tv), d(Tz, Sv), d(Sz, Sv)\}. \\ &\geq \max\{d(Tv, Sv), d(Tz, Tv), d(Tz, Tv), d(Tz, Tv)\}. \\ &\geq d(Tz, Tv). \end{aligned}$$

Taking the upper limit as  $k$  tends to  $\infty$ , we have

$$\begin{aligned} \psi(d(Tz, Tv)) &= \psi(s^2 d(Tz, Tv)) \\ &\leq \psi(\limsup_{k \rightarrow \infty} M_1(z, v)) - \phi(\liminf_{k \rightarrow \infty} N_1(z, v)) \end{aligned}$$



$$\leq \psi(d(Tz, Tv)) - \phi(d(Tz, Tv)),$$

which implies  $\phi(d(Tz, Tv)) = 0$ . It follows that  $d(Tz, Tv) = 0$ .

We see easily that if  $S$  and  $T$  are weakly compatible,  $z$  is a unique common fixed point of  $T$  and  $S$ .

According to Theorem 2.1 we can get the following results.

**Corollary 2.1** Let  $(X, d, s)$  be a complete b-metric like space with constant  $s \geq 1$  and let  $T, S: X \rightarrow X$  be given self mappings with  $T(X) \subset S(X)$ , where  $S(X)$  is closed subset of  $X$ . If the following condition is satisfied:

$$s^2[d(Tx, Ty)] \leq M_1(x, y) - L d(Sx, Sy),$$

where  $L \in (0, 1)$  represents a constant. Then  $T$  and  $S$  have a unique coincidence point in  $X$ . Moreover,  $T$  and  $S$  have a unique common fixed point provided that  $T$  and  $S$  are weakly compatible.

**Corollary 2.2** Let  $(X, d, s)$  be a complete b-metric like space with constant  $s \geq 1$  and suppose  $T, S: X \rightarrow X$  be given self mappings with  $T(X) \subset S(X)$ , where  $S(X)$  is closed subset of  $X$ . If the following condition is satisfied:

$$s^2[d(Tx, Ty)] \leq M_1(x, y) - N_1(x, y),$$

then  $T$  and  $S$  have a unique coincidence point in  $X$ . Moreover,  $T$  and  $S$  have a unique common fixed point provided that  $T$  and  $S$  are weakly compatible.

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