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Scattering Calculation For Single Channel Approximation Using Resonating Group Method

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Abstract: Microscopic theoretical studies of light nuclear systems are essential for deriving unambiguous physical information regarding nuclear structure, interactions from the host of bound state, scattering and reaction problems. In this paper the microscopic nuclear structure model and resonating group method (RGM) has been described as a tool for such microscopic studies. This further helps in studying the scattering of low energy nucleons in the single channel approximation. Resonance calculation is one of the most difficult in the analysis of nucleus and can dominate the prediction accuracy of the nucleus analysis.

Keywords: Light nuclear systems • RGM • Single channel approximation

Introduction

A number of macroscopic and microscopic methods have been used to study the nucleon motion in different nuclei. Microscopic models have also been used to describe the properties of nuclear collective vibrations. These models use few simple reasonable assumptions to yield numerically a number of observed properties of these nuclei in a simple manner.

In the past years several efforts have been made to develop methods for microscopic theoretical studies of N-nucleon systems. One of the most successful microscopic methods for the study of nuclear system where N is not too large is resonating group method (RGM) (Wheeler, 1937), (Tang, 1978), (Okabe, 1984), (Kondo, 2021). The fundamental principle of RGM is based on two assumptions: (i) the average attractive nature of nuclear forces, (ii) there exists in nuclei relatively long-range correlations, due to which nucleons are grouped together in configuration known as clusters (Herzenberg, 1957), (Wildermuth 1966), (Brink, 2008). The nucleons spend fractions of their time in various substructures or clusters in the nucleus. In this method a nucleon-nucleon potential which is capable of explaining reasonably well the nucleon low-energy scattering data is used and Pauli exclusion principle is taken into account by employing fully antisymmetric wave function.

But in the beginning RGM was used only to study the very light nuclear system (Ferring, 1956) $N \leq 8$ which involved two s-shell nuclei in both the incident and outgoing channel. RGM has certain features which distinguish it from other methods namely

(i) It uses entirely antisymmetric wavefunction that is why the Pauli exclusion principle is fully taken

into account.

(ii) It treats correctly the total center-of-mass motion.

(iii) It uses internal structure of interacting clusters.

(iv) It utilized nucleon-nucleon potential which explain reasonably well the two-nucleon low energy

scattering data.

Due to these features RGM calculations are generally rather difficult to perform especially when the number of nucleons involved in the system in large. Indeed, the requirement of antisymmetrization causes insurmountable computational difficulties and therefore the RGM cannot be expected to be useful in studying systems containing more than eight nucleons. With the development of the generator coordinate technique GCT (Griffin, 1957) it become possible to perform scattering calculations for rather larger systems.

Formulation of Single Channel Approximation

Considering the simplest case of two cluster elastic scattering where specific distortion effect is neglected. Let us consider there are two clusters A and B then the projection equation in this approximation can be written as

$$<\delta\Psi_s \mid H - E_T \mid \Psi_s> = 0 \quad \text{---- (1)}$$

Here H is Hamiltonian for the total system and E_T is total energy of the system. This translationally invariant Hamiltonian H for the N nucleon system can be given as

$$H = \sum_{i=1}^{N} \frac{pi^2}{2m} + \sum_{i<j=1}^{N} V_{ij} - T_{cm} \quad \text{---- (2)}$$

The trial wavefunction of the system is written as

$$\Psi_s = A\,[\phi_A\,\phi_B\, x\,(\overline{R})\,\xi\,(\sigma,\tau) \quad \text{---- (3)}$$

To find the set of basis functions $\delta\Psi_s$, Ψ_s is written in parameter form as to find the set of basis functions $\delta\Psi_s$, the wavefunction Ψ_s is written in parameter form as

$$\Psi_s = \int A\,[\phi_A\phi_B\;\xi\,(\sigma,\tau)\;\delta\,(\overline{R}'' - \overline{R})\;\; x\,(\overline{R}'')\,dR'' \quad \text{---- (4)}$$

Where A is antisymmetrization operator, Φ_A and Φ_B are the internal wavefunctions for the internal motion of nucleon in nuclei A and B respectively x(R") describes the relative motion of the two nuclei and is a function of distance $\overline{R}_{"}$ between their center of mass, $\overline{R}_A$ and $\overline{R}_B$. $\xi\,(\sigma,\tau)$ is the spin-isospin function for the system.

Applying projection equation (1) for one channel approximation, and with the help of well-known properties of antisymmetrization operator A is a Hermitian operator, which commutes with the Hamiltonian operator H and satisfies the relation

$$A^2 = (N_A + N_B)!\; A \quad \text{---- (5)}$$

Here N_A and N_B are the number of nucleons in cluster A and cluster B respectively. We can write (1) as:

$$<\phi_A\,(\vec{x})_1\,\phi_B\,(\vec{x})_2\,\xi\,(\sigma,\tau)\,\delta\,(\overline{R}'' - \overline{R})\;\mid H - E_T\mid A\;\{\phi_A\;(\vec{x})_1\,\phi_B\,(\vec{x})_2\,\xi\,(\sigma,\tau)\;x\;(\vec{R})\}> = 0 \text{ --- (6)}$$

The antisymmetrization operator A can be decomposed as

$$A = A'\,(A_A\,A_B) \quad \text{---- (7)}$$

Where, A_A and A_B are the antisymmetrization operators with respect to the nucleons of nucleus A and nucleus B, respectively. A' is the rest antisymmetrization operator which takes care of the remaining permutations of nucleons between nuclei A and B.

Using equation,

$$A' = 1 + (A' - 1) \quad \text{---- (8)}$$

eqn. (7) may be written as

$$<(A_A\Phi_A)\,(A_B\Phi_B)\,\xi\,(\sigma,\tau)\delta\,(\vec{R}'' - \vec{R})\mid H_A + H_B - V' - \frac{h^2}{2u}\;{}_R{}^2\nabla - E_T\mid$$

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$$(A_A\Phi_A)\,(A_B\Phi_B)\,\xi(\sigma,\tau)\,x(\vec{R})> +$$

$$<(\quad A_A \quad \Phi_A) \quad (\quad \nabla A_B \quad \Phi_B)$$

$$\xi(\sigma,\tau)\delta(\vec{R}'' - \vec{R})|\,H_A + H_B - V' - \frac{h^2}{2u}\,{}_R{}^2 - E_T|$$

$$(A' - 1)\{(A_A\Phi_A)\,(A_B\Phi_B)\,\xi(\sigma,\tau)\,x(\vec{R})\}> = 0 \qquad \text{---- (9)}$$

As, the operator $(-\frac{h^2}{2u}\,{}_R\nabla^2)$ and V' do not act on the co-ordinates $\vec{x}_1$ and $\vec{x}_2$ and operators H_A and H_B act only on $\underline{x}_1$ and $\underline{x}_2$, respectively, the first matrix element of equation (2.24) can be easily obtained, after summing over all spin – isospin co-ordinates and integrating overall spatial internal co-ordinates, and $\underline{R}$'', it gives

$$\{-\frac{h^2}{2u}\nabla_R{}^2 - E + V_D(\vec{R})\}\,x(\vec{R}) \qquad \text{---- (10)}$$

where, E is the energy of relative motion of the nuclei A and B and is given by

$$E = E_T - (A_A\Phi_A\,|\,H_A|A_A\Phi_A) - (A_B\Phi_B\,|\,H_B\,|\,A_B\Phi_B) \qquad \text{---- (11)}$$

Here, the round brackets denote integration overall internal co-ordinates of A and B. $V_D(\vec{R})$ is the direct potential and is given by

$$V_D(\vec{R}) = <(A_A\Phi_A(\vec{x}_1))\,(A_B\Phi_B(\vec{x}_2))\,|V'|\,(A_A\Phi_A(\vec{x}_1))\,(A_B\Phi_B(\vec{x}_2))> \text{---- (12)}$$

The second term of equation (9) can be solved easily by replacing $x(\vec{R})$ by

$$x(\vec{R}) \sim \int x(\vec{R})\,\delta(\vec{R} - \vec{R}')\,d\vec{R}' \qquad \text{---- (13)}$$

Thus, it becomes

$$\int <(A_A\Phi_A(\vec{x}_1)\,(A_B\Phi_B(\vec{x}_2)\xi(\sigma,\tau)\delta(\vec{R}-\vec{R}'')|H|\,(A'-1)$$
$$\{(A_A\Phi_A(\vec{x}_1)(A_B\Phi_B(\vec{x}_2)\delta(\vec{R}-\vec{R}')\xi(\sigma,\tau)\}x(\vec{R}')>\,d\vec{R}'$$

$$-\ E_T \int <(A_A\Phi_A(\vec{x}_1)\,(A_B\Phi_B(\vec{x}_2)\xi(\sigma,\tau)\delta(\vec{R}-\vec{R}'')|\,(A'-1)$$

$$\{\quad (A_A\Phi_A(\vec{x}_1)(A_B\Phi_B(\vec{x}_2)\xi(\sigma,\tau)\delta(\vec{R}-\vec{R}')\}x(\vec{R}')>\,d\vec{R}' \qquad \text{----- (14)}$$

which may be written, after carrying out the delta function integration over $\underline{R}$'', as

$$\int K(\vec{R},\vec{R}')\,x(\vec{R})\,d\vec{R}' \qquad \text{---- (15)}$$

where, $K(\vec{R},\vec{R})$ is the non-local potential.

Thus, from equations (15) and (10), the projection equation (1) can be finally written as

$$(\frac{h}{2\mu}\nabla_R{}^2 + V_D(\vec{R}) - E)\,x(\vec{R}) + \int K(\vec{R},\vec{R}')\,x(\vec{R})\,d\vec{R}' = 0 \qquad \text{---- (16)}$$

The above equation is the integrodifferential equation for the scattering function $x(\vec{R})$ in (RGM) for single channel approximation.

Conclusion

It can be seen that in the case of RGM, the integrodifferential equation for the scattering function in the scattering of two composite nuclei in the single channel approximation is difficult to solve when the same is applied to the systems with A > 8. As in RGM the nucleon-nucleon potential is highly non-local and energy dependent due to the antisymmetrization with respect to all nucleons. The task of solving the Kernels is a difficult problem. To overcome this difficulty

for the systems with A > 8, we can consider Generator Coordinate Method (GCM). The GCM is a variational method (Beck, 1975) based on a many-body nuclear Hamiltonian. This method leads to the development of an integral equation describing the collective motion of a many-nucleon system and is considered as a powerful tool to study collective vibrations in nuclei.

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