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A Nuclear Electric Quadrupole Moment Of ^{6}Li Nucleus Using Hyperfine Structure In Atomic Spectra

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Abstract: The nuclear electric quadrupole moment (QM) is one of the most important bulk properties of the nucleus with which nuclear deformations can be investigated. In this paper the electric quadrupole moment for ^{6}Li nucleus in the ground state is calculated from the hyperfine structure of atomic spectra. Initially a cluster model wavefunction is constructed in shell model using Resonating Group Method (RGM), Generate coordinate method (GCM) and Complex Generator Coordinate Technique (CGCT). CGCT transforms the cluster model wavefunction into antisymmetrized product of single particle wavefunction.The wavefunction is written with the total angular momentum, relative motion between alpha and deuteron clusters, definite parity and spin. The calculations have been made by taking Wood- Saxon potential. The results obtained are compared and found reasonably well in agreement with the experimental results.

Key words: RGM, GCM, CGCT, Wavefunction, Quadrupole moment

Introduction

The nuclear electric quadrupole moment (EQM) measures the deviation of charge distribution (Schelur H. 1936) from a spherical shape and hence provide information about nuclear shape. The measurement of EQM involves interaction of the nuclear charge distribution with the static charge distributions of the electrons in atomic and molecular systems or with a specified external applied electric field.

There are different ways to determine the nuclear shape, but here we have considered the interaction (Meyer U. 1959) of the nuclei with fast charged particles i.e.,the electron scattering. In electron scattering experiments, the measurement of scattering cross section is interpreted in terms of a charge distribution over a finite radius.

Formulation of Electric Quadrupole moment

The atomic nuclei can be considered as a small electric charge and magnetic moment distributions. The electrostatic energy of a localized charge distribution described by charge density $\rho(x)$ placed in an external potential (Jackson J.D. 1962) $\Phi(x)$ is

$$W = \int \rho(x)\Phi(x)dx^3 \qquad (1)$$

Since the nucleus is of a small dimension the potential $\Phi(x)$ will change by very small amount over the nuclear volume and $\Phi(x)$ can be explained in a Taylor's series around a suitably chosen origin

$$\Phi(x) = \Phi(0) + \bar{x}.\nabla\Phi(0) + \frac{1}{2}\sum_{ij} x_i . x_j . \frac{\partial^2\Phi(0)}{\partial z_i \partial x_j} + --- \quad (2)$$

Substituting E = $-\nabla\Phi$ and since $\nabla . E = 0$

forthe extended field, one can subtract $\frac{1}{6} r^2 \nabla . E(0)$ from the eq. (2)

$$\Phi(x) = \Phi(0) - \bar{X}.\bar{E}(0) - \frac{1}{6}\sum_{ij}(3x_i x_j - r^2\delta_{ij})\frac{\partial E_j(0)}{\partial x_j} + --- \quad (3)$$

In this equation the first term represents the potential at x if the total charge were concentrated at the origin and is called the potential due to monopole moment of charge distribution. The second term represents the potential which would result if a point dipole having dipole moment equal to that of the charge distribution were located at the origin of co-ordinates and is called potential due to dipole moment of the charge distribution. The third term is called the potential due to quadrupole moment of charge distribution and other higher order terms are called multipole potentials. The Quadrupole moment tensor Q_{ij} is given by

$$Q_{ij} = \int (3x_i x_j - r^2\delta_{ij})\,\rho(x) d^3x \quad (4)$$

Since the wavefunction of a stationary state has a definite parity, because of symmetry inversion for ρ(x) the nuclear electric dipole moments are essentially zero. The ρ(x) of the nucleus is cylindrically symmetric about the z-axis. Hence the only non-vanishing quadrupole moment tensor component is Q_{33}.

The quantum mechanical expressions for the operators corresponding to total charge, dipole moment and quadrupole moment are obtained by substituting the following

$$\int \rho(r) d^3r = \int Z e\,|\varphi(r_1 r_2 ------ r_A)|^2 d^3r_1 d^3r_2 --- d^3r_A \quad (5)$$

where, $\psi(r_1 r_2 ----- r_A)$ is the nuclear wavefunction, r_1 r_2 ----- r_A are position coordinates of 'A' nucleons in the nucleus.

The expression for quadrupole moment

$$Q_{ij} = \sum_{K=1}^{Z} e\,((3x_i x_j - \delta_{ij} r^2)_K\,|\Psi(\bar{r_1} ---- \bar{r_A})|^2 d^3r_1 --- d^3r_A \quad (6)$$

From eq. (6) the quadrupole moment is the expectation value of the operator $\frac{1}{e}$ (Q_{33}) in the nuclear state Φ_{JM} with M = J where e is the proton charge.

$$Q_{JM} = \frac{\frac{1}{e}\langle\Psi_{JM}|\int(3z^2 - r^2)\rho(\bar{r})d^3r|\Psi_{JM}\rangle}{\langle\Psi_{JM}|\Psi_{JM}\rangle} \quad (7)$$

The unit of quadrupole moment is length^2.

Calculation of Electric Quadrupole Moment

For the calculation of electric quadrupole moment of ^{6}Li nucleus in ground state first we have written the cluster shell model wavefunction (Sinha P. 2020) by assuming that it is made up of one alpha (2 protons and 2 neutrons) cluster and one deuteron (1proton and 1neutron) clusterdue to long range correlation between protons and neutrons. The spin and parity of the nucleus in ground state is $J^\Pi = 1^+$. The shell model emanates from an average potential with a shape something between a square well and a harmonic oscillator (Tang 1977). In this case we have used a more realistic but at the same time a complicated potential known as Wood-Saxon potential. The integral representation for the wavefunction Φ for the ground state of ^{6}Li nucleus is

$$\Phi_{1M} = A \int \Phi_S(\alpha, R'_\alpha)\Phi_S(d, R'_\alpha)\, \delta(R_\alpha - R'_\alpha)\, \delta(r_d - r'_d)\xi_\alpha \xi_d x_{l_{ml}}(R'_\alpha - r'_d) Z_{RCM}\left(\frac{4R'_\alpha + 2r'_d}{6}\right). dR'_\alpha\, dr'_d \quad (8)$$

substituting eq. (8) in eq. (7) and solving it by fitting the value of width parameters of alpha cluster and deuteron cluster, the calculated value of quadrupole momentis - 0.001283 barns.

Results and Discussion

In the present calculations the value of quadrupole moment has been calculated as-0.001283 barns. We have fitted the value of alpha and beta parameters from the binding energy calculations for the ground state of ^{6}Li nucleus. The present value of QM is compared with the theoretical calculation (Xiao. 2018) which is -0.00081 barns and that with one more theoretical calculation (James C, 1998) where the resultis -0.8178 barns. Our results are in good agreement with the experimental value (Raghavan P, 1989) where QM is measured as-.00083 barns.The technique of writing wavefunction is already used for the calculating different parameters for ^{5}He and^{6}Li nucleus in the ground state. The wavefunction was employed to calculate the charge form factor for ^{5}He nucleus (Sinha P. 2019) and the root mean square radius for ^{6}Li nucleus (Sinha N. 2020) and the same were ingood agreement with the experimental data. We can further use the presently developed wavefunction to calculate charge form factor for ^{6}Li as well.

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