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Root Mean Square Radius of ^{6}Li Nucleus using Cluster Model Wavefunction

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Abstract: The calculation of root mean square nuclear charge radius is one of the most important nuclear parameters regarding the size and structure of the nucleus. In this paper calculations for root mean square (r.m.s.) radius of the ground state of ^{6}Li nucleus using high energy electron scattering is presented. The initial work involves writing first the cluster model wavefunction employing the resonating group method, generator coordinate method and complex generator coordinate technique. The wavefunction is written with definite parity, spin, total angular momentum and relative motion between the alpha cluster and deuteron cluster and the center-of-mass of the two clusters. The application of complex generator coordinate technique transforms the cluster model wavefunction into antisymmetrized products of single particle wavefunction written in terms of single particle co-ordinates, the center-of-mass coordinates, parameter coordinates and generator coordinates. The width parameters of alpha and deuteron clusters are adjusted to obtain predictions close to experimental values.

Keywords: clusters, wavefunction, ^{6}Li nucleus

Introduction

The measurement of rms radius of the light nuclei is of great interest since it serves as a good criteria for the correctness of the wavefunction chosen for the ground state of the nuclei .The root mean square charge radius can be determined by the two methods ,one is from the scattering experiments with high energy electrons (Hofstadter 1956) and second possible method is the experiment in which the total effective cross section for elastic dipole absorption of photons by the nucleus is measured .

The determination of the root mean square charge radius (Sens, 1967) by means of elastic scattering for light nuclei in itself is superior and trust worthy to other means of investigations for a variety of reasons. One principal reason is that the interaction between electron and nucleus is well known to a high degree of accuracy, as electron being negatively charged is deflected when it strikes the nucleus according to how the charge is distributed inside the nucleus. I have written ^{6}Li wavefunction using complex generator coordinate technique in my earlier paper (Sinha et.al 2020).

Formulation of Charge RMS Radius

The ground state of 6 Lithium nucleus consists of three protons and three neutrons. Due to long-range correlation these protons and neutrons are grouped together and form one Alpha cluster (consisting of two protons and two neutrons) and one deuteron cluster (consisting of one proton and one neutron).

The expression for rms radius of ^{6}Li nucleus is given as (Okamoto et al. 1974)

$$\langle R_{ch}^2\rangle^{1/2} = \langle R_m^2\rangle^{1/2} + \langle R_p^2\rangle^{1/2} \quad (1)$$

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where $<R_p^2>$ is the mean square proton radius and its numerical value is 64fm^2.

$<Rm^2>$ is the mean square matter radius which gives the overall matter distribution, proton plus neutrons i.e., it represents nucleus as a whole and is given by

$$<R^2{}_m> = \frac{\frac{1}{6}<\Phi_1 M \mid \sum_{i=1}^{6}(\bar{r}-\bar{R}cm)^2 \mid \Phi_1 M>}{<\Phi_{1M} \mid \Phi_{1M}>_R} \qquad (2)$$

In equation (2) $<>'_R$ represents integral over the relative coordinates and summation over spin and isospin coordinates. We may write the quantity $\sum_{i=1}^{6}\overline{(r - Rcm)}^2$ for convenience as follows

$$\frac{1}{6}\sum_{i=1}^{6}\bar{r}_i - \bar{R}cm \qquad (3)$$

and $$\sum_{i=1}^{6}\overline{(r_i - Rcm)}^2 = r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + R_{cm}^2 - 2\bar{R}_{cm}(\bar{r}_1 + \bar{r}_2 + \bar{r}_3 + \bar{r}_4 + \bar{r}_5 + \bar{r}_6) \qquad (4)$$

substituting eq. (3) is eq. (4) we have

$$\sum_{i=1}^{6}\overline{(r_i - Rcm)}^2 = r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + 6\,R_{cm}^2 \qquad (5)$$

using eq. (5), we can write eq. (2) as

$$<R_m^2> = \frac{\frac{1}{6}<\Phi_{1M} \mid \sum_{i=1}^{6} ri^2 \mid \Phi_{1M}>}{<\Phi_{1M} \mid \Phi_{1M}>} - \frac{<\Phi_{1M} \mid R_{cm}^2 \mid \Phi_{1M}>}{<\Phi_{1M} \mid \Phi_{1M}>} \qquad (6)$$

the second term in eq. (6) may be written as

$$< R_{cm}^2> = \frac{\int 1\, z(\bar{R}cm)|^2 . R_{cm}^2\, d\,\bar{R}cm}{\int 1\, z(\bar{R}cm)|^2} \qquad (7)$$

on solving the above integral, we get

$$<R^2{}_{cm}> = \frac{1}{32}$$

Substituting all these values in eq. (1) we get

$$<R^2{}_{ch}>^{1/2}{}_{6Li} = 2.18 \text{ fm}$$

Conclusion

In this paper the calculated values of charge root mean square radius for the ground state of ^{6}Li nucleus is 2.18 fm. We have added the correction term for the proton radius as .64 fermi in my formulation. The present value of charge rms radius is in the good agreement with the experimental value as 2.09 $\pm$ 0.02 fm (Tanihata et. al. 1985). This technique of writing wavefunction is already used for ^{5}He nucleus (Sinha, 2013) and ^{9}Be (Sinha, 2011). The Calculation of charge form factor for ^{5}He(Sinha et. al. 2019) nucleus was in a good agreement with the experimental data. We can further use the presently developed ^{6}Li wavefunction to calculate quadrupole moment and charge form factor. The complex generator technique is good enough as it converts cluster model wavefunction into anti-symmetrized product of single particle wavefunction, which makes the evaluation of matrix elements less lengthy that arisein the calculation of normalization constant, rms radius, quadrupole moment, charge form factor.

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