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Some Properties on Divine Kaehlerian Manifold

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Abstract: Fibonacci sequence and the divine ratio are intimately inter-connected. In the Fibonacci sequence each number is the sum of previous two consecutive numbers and the ratio of any two consecutive numbers reflects the approximate value of divine ratio. The relationship between divine ratio and Fibonacci series is well express in divergent faunal anatomy and floral as well as their morphology. The present article is intended to study the properties of divine Kaehlerian manifold in terms of Fibonacci sequence, trace & eigen values of divine structure including its almost complex structures. Some properties of induced structures, theorems and propositions related to it have also been studied.

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1. Introduction

Our nature is really beautiful adding mathematics with divergent branches of science. The Fibonacci has an apparent contact with the Statistics; Operational Research and Computational mathematics covering divergent geometric topics for example divine section etc. The ideas of Fibonacci number are widely relevant to growth of every living thing, involving a cell, from a wheat grain to a bee's hive. The universe may be unsettled and uncertain, but it also an overmuch unified physical region bound by mathematics laws. From the primitive times Fibonacci numbers attracts mathematics for their unique beauty and abounded notions possessing a unique feature with their notions in divergent region of science unrelated to mathematics (Sarma and Bhuyar, 2018). Golden ratio or golden section or divine proportion is those number which is approximately equal to 1.618 this number is also called an irrational mathematical constant 'phi' and is denoted the letter φ from the Greek alphabet. In the easiest form Divine ratio is the division of a line into an exclusive ratio producing an artistically attractive fraction, keeping a pathetic usage in architecture and art. Divine ratios were also reported to be present in divergent plant anatomy as well as human body parts (*http://en*.).

A Kaehler manifold as well as complex manifold (Y) that is closed with 2-form and has a hermitian matrix h. In greater detail, at each point of complex manifold, h gives a positive definite hermitian form on the tangent space and the **w** 2-form such that:

$$w(u,v) = Re\ h(iu,v) = im\ h(u,v)$$

For vector tangent **u** and **v** (where i is the complex number $\sqrt{-1}$) for a Kaehler manifold, the Kaehler form **w** is real and closed to 1, 1-form. A Kaehler manifold can also be viewed as a Riemannian manifold (R) with Riemannian metric g such that:

$$g(u,v) = Re\ h(u,v)$$

A Kaehler manifold is a hermitian manifold of complex dimension n, such that for every point Q of complex manifold, there is a holomorphic co-ordinate chart around r, in which the metric on C^n is represented in these co-ordinates as:

$h_{ab} = \left(\frac{\partial}{\partial z_a}, \frac{\partial}{\partial z_b}\right) h_{ab} = \left(\frac{\partial}{\partial z_a}, \frac{\partial}{\partial z_b}\right)$, then $h_{ab} = \delta_{ab} + 0(\|z\|^2) h_{ab} = \delta_{ab} + 0(\|z\|^2)$

For all a, b lies from 1, 2, 3, …….n.

The 2-form is closed in this case, and thus determines the Kaehler class in De-Rham cohomology $H^2(Y, R)$. A Kaehler manifold is an even-dimensional Riemann manifold X with a holonomy group contained in the unitary group $u(n)$. In addition, at each point on the tangent space of complex manifold (Y), there is a complex structure J (which is a real linear map from T_y and J^2 is equal to negative one) and maintains the metric g. As a result, J preserves parallel transport and the alternating two-form is as follows:

$$w(Y, X) = g(JY, X)$$

Which shows that, if **w** is closed then g is a Kaehler metric in Kaehler manifold N.

2. Divine Kaehlerian Manifold

In this part we prescribe the structure of polynomial in a n-dimensional Kaehlerian manifold. The structure represented by $(\tilde{N}, \tilde{h})$ called divine structure, which determine by $1, 1 -$ type tensor field $\tilde{Q}$. It is sufficient for the equation

$$\tilde{Q}^2 = \tilde{Q} + I, \tag{2.1}$$

where, identity operator (I) is on the Lie derivative $\eta(\tilde{N})$ of vector fields N. Here, the hermitian matrix $\check{h}$ in $\tilde{Q}$ is applicable, if the similarity

$$\tilde{h}(\tilde{Q}(W), X) = \tilde{h}(W, \tilde{Q}(X)) \tag{2.2}$$

It is sufficient, for every vector field of tangent $W, X \in \eta(\tilde{N})$.

***Note:** For a divine structure $\breve{Q}$, Kaehlerian manifold $(\tilde{N}, \tilde{h})$ is equivalent with*

$$\tilde{h}\left(\tilde{Q}(W), \tilde{Q}(X)\right) = h\left(W, \tilde{Q}(X)\right) + \tilde{h}(W, X) \tag{2.3}$$

$\forall$ tangent vector fields $U, V \in \eta(\tilde{N})$.

Theorem 2.1: *A divine Kaehlerian manifold $(\tilde{N}, \tilde{h}, \tilde{Q})$ has the property*

$$\tilde{Q}^m = \varphi_m \tilde{Q} + \varphi_{m-1} I \tag{2.4}$$

$\forall$ integer number m>0, where $(\varphi_m)_m$ is the Fibonacci sequence.

Proof: If we suppose

$\tilde{Q}^n = \varphi_n \tilde{Q} + \varphi_{n-1} I$ $(n > 0)$, we get

$\tilde{Q}^2 = \tilde{Q} + I\tilde{Q}^2 = \tilde{Q} + I$; $\tilde{Q}^3 = 2\tilde{Q} + I$

and $\tilde{Q}^4 = 3\tilde{Q} + \tilde{Q}^{n+1} = \varphi_n \tilde{Q}^2 + \varphi_{n-1}\tilde{Q}$

$= (\varphi_n + \varphi_{n-1})\tilde{Q} + \varphi_n I$

Hence the result.

Remark: A Kaehlerian manifold $(\tilde{N}, \tilde{h})$ with a divine manifold $\tilde{Q}$ is signify by divine Kaehlerian structure and $(\tilde{h}, \tilde{Q})$ itself a divine Kaehlerian structure on N.

Theorem 2.2 *The trace and Eigen values of the divine structure $\tilde{Q}$ defined in an n-dimensional Kaehlerian manifold $(\tilde{h}, \tilde{Q})$ are the divine ratio $trace(\tilde{Q}^2) = trace(Q) + n$ and the eigen values are Ψ and $(\Psi - 1)$.*

Proof: Let $\{f_1, f_2 \ldots\ldots\ldots\ldots f_n\}$ be an orthonormal basis of the tangent space $T_y\tilde{N}$ on each point of $y \in \tilde{N}$. Now, from eq. (2.1), we get

$$\tilde{h}(\tilde{Q}^2 f_i, f_i) = \tilde{h}(\tilde{Q} f_i, f_i) + \tilde{h}(f_i, f_i)$$

Summing it by i, we have

$$trace(\tilde{Q}^2) = trace(Q) + n \tag{2.5}$$

Further, if λ is an eigen value of the divine structure $\tilde{Q}$ on $T_y\tilde{N}(\forall\, y \in \tilde{N})$ then $\tilde{Q}X = \lambda X$ for all vector fields of tangent $X \in T_y\tilde{N}$. At each point of $y \in \tilde{N}$, we get $\lambda^2 = \lambda + I$,

which follows that eigen values of $\tilde{Q}$ are divine ratio $\lambda_1 = \Psi$ and $\lambda_2 = I - \Psi$.

Remark: If Kaehlerian manifold $\tilde{N}$ is an n-dimensional completed with a positive definite Kaehlerian hermitian matrix $\tilde{h}$ and suppose that a non-trivial field of tensor K of $1,1$ –type such that $K^2 = I$ and $\tilde{h}(Ku, Kv) = \tilde{h}(u, v)$ for all vector fields $u, v \in \eta(\tilde{N})$, then K is known as an almost product structure and $(\tilde{N}, \tilde{h}, \tilde{K})$ represent as an almost product Kaehlerian manifold.

Proposition 2.3 *Each almost product structure K in an n- dimensional Kaehlerian manifold $(\tilde{N}, \tilde{h})$ induced to divine structures on $(\tilde{N}, \tilde{h})$ as:*

$$\tilde{Q}_1 = \frac{I + \sqrt{5}K}{2},\ \tilde{Q}_2 = \frac{I - \sqrt{5}K}{2}$$

vice-versa. Also, the divine structure $\tilde{Q}$ prescribed on a Kaehlerian manifold $(\tilde{N}, \tilde{h})$ induced with an almost product structure of this manifold.

Proof: Let almost product structure K prescribed in an n-dimensional Kaehlerian manifold $(\tilde{N}, \tilde{h})$ by using a divine structure $\tilde{Q}$ in the form $K = a\tilde{Q} + bI$, where $a, b \in R^*$, thus

$$K^2 = a^2\tilde{Q}^2 + 2ab\tilde{Q} + b^2 I$$

using that $K^2 = I$ and $\tilde{Q}^2 = \tilde{Q} + I$, we get

$$\tilde{Q}_1 = \frac{I + \sqrt{5}K}{2},\ \tilde{Q}_2 = \frac{I - \sqrt{5}K}{2}.$$

Moreover,

$$\tilde{h}(\tilde{Q}_i(W), X) = \tilde{h}\left(W, \tilde{Q}_i(X)\right) \Leftrightarrow \tilde{h}(\tilde{Q}(U), V) = \tilde{h}(U, \tilde{Q}(V))$$

$\forall\, i \in \{1,2\}$ and vector fields of tangent $W, X \in \eta(\tilde{N})$. On a divine Kaehlerian manifold $(\tilde{N}, \tilde{h}, \tilde{Q})$, we can prescribe two operators' projection

$$l = \frac{1}{\sqrt{5}}(\Psi I - \tilde{Q})$$

and

$$m = \frac{1}{\sqrt{5}}(\Psi - I)(I + \tilde{Q})$$

3. Properties of induced structures on sub-manifolds in $(\tilde{N}, \tilde{h}, \tilde{Q})$

Let N be an n-dimensional sub-manifold, absorbed on a Kaehlerian manifold $(\tilde{N}, \tilde{h}, \tilde{Q})$ with a Kaehlerian hermitian matrix $\tilde{h}$ and a divine structure $\tilde{Q}$ such that the matrix $\tilde{h}$ is $\tilde{Q}$ relevant. We denote by T_yN the tangent space of N on $y \in \tilde{N}$ and by $T_y{}^{\perp}N$ the normal space of N in y, for every $y \in \tilde{N}$. Let i_* be the immersion differential $i: N \rightarrow \tilde{N}$.

The induced Kaehlerian hermitian matrix h of N is given by

$$h(W, X) = \tilde{h}(i_*W, i_*Y)\ for\ all\ W, X \in \eta(\tilde{N}) \qquad (3.1)$$

Let us assume an orthonormal basis $\{f_1, f_2, \dots f_n\}$ of the normal space $T_y(N)^{\perp}$ at each point of $y \in \tilde{N}$. Let us suppose that range of indices a, b and c are from 1, 2...n and $a, b, c \in \{1, \dots, n\}$.

For any $W \in T_yN$, $\tilde{Q}_{i_*}W$, $\tilde{Q}M_a$ can be decomposed in normal components and tangential at N in the form:

$$\tilde{Q}_{i_*}W = i_*QW + \sum_a V_a(W)M_a\ \forall\, W \in \eta(N) \qquad (3.2)$$

$$and \qquad \tilde{Q}M_a = \varepsilon i_*\mu_a + \sum_b e_{ab}M_b \quad (\varepsilon = \pm 1), \qquad (3.3)$$

where Q is an $1,1$ $-$type tensor field, μ_a are vector fields of tangent on sub-manifold N, V_a are 1-forms on N and $e := (e_{ab})_n$ is a real function $n \times n$ matrix on N. Thus, we get a structure $(Q, h, V_a, \mu_a. (e_{ab})_n)$ inspired on N by $(\tilde{Q}, \tilde{h})$ from the gauss and Weingarten formulae are:

$$\tilde{\nabla}_W X = \nabla_W X + \sum_{a=1}^{n} j_a(W, X)M_a \qquad (3.4)$$

$$\tilde{\nabla}_W M_a = -B_aW + \nabla_W^{\perp} M_a, \qquad (3.5)$$

where

$$j_a(W, X) = h(B_aW, X), \forall\, W, X \in \eta(N).$$

If $\{f_1, f_2, \dots f_n\}$ and $\{f_1', f_2' \dots f_n'\}$ are two orthonormal bases on a normal space $T_y{}^{\perp}N$ then the decomposition of f_a' in the base $\{f_1, f_2, \dots f_n\}$ is the following

$$F_a' = \sum_{c=1}^{n} l_a^c M_c \qquad (3.6)$$

For any $a \in \{1 \dots n\}$, where $(l_a{}^c)$ is an $n \times n$ orthogonal matrix and we have $V_a' = \sum_c l_a^c V_c$,

$$\mu_a' = \sum_c l_a^c \mu_c\ and\ e_{ab}' = \sum_c l_a^c e_{cd} l_b^d.$$

Thus, if $\mu_1 \dots \mu_n$ are vector fields which is linearly independent, then $\mu_1' \dots \mu_n'$ are also linearly independent further because e_{ab} is symmetric in a and b under a suitable transformation, we can find that e_{ab} can be reduce to $e_{ab}' = \lambda_a d_{ab}$, where λ_a $(a \in \{1 \dots n\})$ are eigen values of the matrix $(e_{ab})_n$ and in this case we have,

$$V_a'\mu_a = \varepsilon d_{ab}(1 + \lambda_a - \lambda_a\lambda_b)$$

and from this we get

$$V_a'\mu_a = \varepsilon(1 + \lambda_a - \lambda_a^2)V_a'\mu_a = \varepsilon(1 + \lambda_a - \lambda_a^2).$$

Proposition 3.1 *if N is an n-dimensional sub-manifold of co-dimension n, in a divine Kaehlerian manifold* $(\tilde{N}, \tilde{h}, \tilde{Q})$, *then the structure* $(Q, h, V_a, \varepsilon\mu_a, (e_{ab})_n)$ *induced on M by the structure* $\tilde{Q}$ *has the following properties:*

i. $(\nabla_W Q)(X) = \mathcal{P}(W, X) + \varepsilon \sum_a j_a(W, X)\mu_a + \sum_a V_a(X)B_aW$

ii. $(\nabla_W V_a)(X) = \tilde{h}(\mathcal{P}(W,X), M_a) - j_a(W, QX) + \sum_b (V_b(X) m_{ab}(W) + j_b(W,X) e_{ba})$

iii. $\nabla_W \mu_a = \mathcal{P}(W, M_a)^T - \varepsilon Q(B_a W) + \varepsilon \sum_b e_{ab} B_b W + \sum_b m_{ab}(W) \mu_b$

iv. $W(e_{ab}) = \tilde{h}(\mathcal{P}(W, M_a), M_b) - \varepsilon V_a(B_b W) - V_b(B_a W) + \sum_c [m_{ac}(W) e_{cb} + m_{bc}(W) e_{ac}]$

for any $W, X \in \eta(N)$

Proof: From the equation

$$\tilde{\nabla}_W(\tilde{Q}X) = \nabla_W QX - \sum_a V_a(X) B_a W + \sum_a [m_a(W, QX) + W(V_a(X) + \sum_b V_b(X) m_{ba}(W)] M_a$$

and

$$\tilde{Q}(\tilde{\nabla}_W X) = Q(\nabla_W X) + \varepsilon \sum_a j_a(W,X)\, \mu_a + \sum_a [V_a(\nabla_W X) + \sum_b j_b(W,X) e_{ba}] M_a,$$

we get

$$\mathcal{P}(W,X) = (\nabla_W Q)(X) - \sum_a V_a(X) B_a W - \varepsilon \sum_a j_a(W,X) \mu_a + \sum_a [j_a(W, QX) + (\nabla_W V_a)(X) + \sum_b V_b(X) m_{ba}(W) - \sum_b j_b(W,X) e_{ba}] M_a.$$

Thus, the tangential part is identifying and respectively the normal part on the last similarity, we get (i) and (ii) property

$$\tilde{\nabla}_W(\tilde{Q}M_a) = \varepsilon \nabla_W \mu_a - \sum_b e_{ab} B_b W + \sum_b [W(e_{ab}) + \varepsilon j_b(W, \mu_a) + \sum_c e_{ac}.m_{cb}(W)] M_b$$

Also, from

$$\tilde{Q}(\tilde{\nabla}_W M_a) = -Q(B_a W) + \varepsilon \sum_b m_{ab}(W) \mu_b - \sum_b [V_b(B_a W) - \sum_c e_{cb} m_{ac}(W)] M_b,$$

we get

$$\mathcal{P}(W, M_a) = \varepsilon \nabla_W \mu_a + Q(B_a W) - \varepsilon \sum_b m_{ab}(W) \mu_b - \sum_b e_{ab} B_b W + \sum_b [W(e_{ab}) + \varepsilon j_b(W, \mu_a) + V_a(B_a W) - \sum_c e_{cb} m_{ac}(W) - e_{ac}.m_{cb}(W)] M_b$$

Thus, the tangential part is identifying and respectively the normal part on the last equality, we get (iii) and (iv)

Proposition 3.2 *Let N be a n- dimensional sub-manifold of co-dimension n in a divine Kaehlerian manifold* $(\tilde{N}, \tilde{h}, \tilde{Q})$ *with* $\tilde{\nabla}\tilde{Q} = 0$ *if* $(Q, h, V_a, \mu_a, (e_{ab})_n)$ *is the induced structure on N by* $(\tilde{Q}, \tilde{h})$ *and* ∇ *is the levi-civita connection defined on N with respect to h then the Nijenhuis tensor field of Q has the form:*

$$N_Q(W,X) = -\sum_a h\left((QB_a - B_a Q)(W), X\right)\mu_a - \sum_a h(X, \mu_a)(QB_a - B_a Q)(W) + \sum_a (W, \mu_a)(QB_a - B_a Q)(X)$$

For any $W, X \in \eta(N)$.

Remark: N be a n-dimensional sub-manifold of co-dimension m in a divine Kaehlerian manifold $(\tilde{N}, \tilde{h}, \tilde{Q})$ and $(Q, h, V_a, \mu_a, (e_{ab})_n)$ be the induced structure on N by $(\tilde{Q}, \tilde{h})$ if $\tilde{\nabla}\tilde{Q} = 0$ and (1,1) tensor field Q on N commutes with the operators of Weingarten

$$B_a\big(i.e\ (QB_a = B_a Q)\big)\ for\ any\ a \in \{1, \dots, m\}$$

then the Nijenhuis tensor field of Q vanishes on N i.e., $(M_Q(W, X) = 0)$ for any $W, X \in \eta(N)$.

Theorem 3.3 *Let N be an n-dimensional non-invariant sub-manifold of co-dimension m immersed in a divine Kaehlerian manifold* $(\tilde{N}, \tilde{h}, \tilde{Q})$ *so that vector fields of tangential* $\eta_1, \eta_2, \dots, \eta_m$ *are linearly independent. Then*

$$trace\ (Q) = m - tr(B) + \sum_{A=m+1}^{n} \lambda_B, m < n$$

$and\ m - tr(B), n = m$ with

$$\lambda_B \in \{\Psi, (1-\Psi)\}\ \forall A, B \in \{m+1, \dots, n\}.$$

Proof: Let matrices (Q) (of Q) $V := \{\mu_1, \mu_1, \dots, \mu_m\}$ and $B := (e_{ab})_m$.

Let us assume that

$$(Q)V = V(I_m - B), when\ I_m = (\delta_{ab})$$

is the identity matrix of order m.

i. For

$$m = n, from\ |V| \neq 0\ we\ get\ (Q) = V(I_m - B)V^{-1},$$

and from this, we have

$$Q_a^b = \sum_{\rho,\sigma} u_\rho^b (\delta_\sigma^\rho - e_\sigma^\rho) v_a^\sigma.$$

Here $a, b, \rho, \sigma\ lies\ from\ 1,2, \dots, m.$ Also $Q_a^b, u_\sigma^b\ and\ v_a^\rho$ are the components of matrices $(Q), V\ and\ V^{-1}\ respectively,$

hence, we have

$$tr(Q) = m - tr(B).$$

ii. For $m < n$

Suppose the matrices

$$\bar{V} and\ O\ by{:}\ \bar{O}$$

$$\bar{V} = (\mu_1, \mu_2, \dots, \mu_m, \dots \eta_{m+1} \dots \eta_n)\ and\ O = \begin{pmatrix} \delta_{ab} - e_{ab} & 0 \\ 0 & \lambda_B \delta_{BA} \end{pmatrix},$$

where

$$A, B\ lies\ from\ m+1, \dots, n A, B\ lies\ from\ m+1, \dots, n.$$

Here

$$\delta_{aa} = 1, \delta_{ab} = 0\ for\ a \neq b\ and\ \lambda_B \in \{\Psi, (1-\Psi)\}$$

are solutions of the equation

$$\lambda^2 = \lambda + 1\lambda^2 = \lambda + 1\ for\ B \in \{m+1, \dots, n\}\ \text{as}\ |\bar{V}| \neq 0 |\bar{V}| \neq 0.$$

Now, from equation

$$(Q) = \bar{V} O \bar{V}^{-1} (Q) = \bar{V} O \bar{V}^{-1},$$

we get

$$Q_a^b = \sum_{\rho,\sigma} \bar{u}_\rho^b l_\sigma^\rho \bar{v}_a^\sigma (a,b,\rho,\sigma) \in \{1,2,\dots,m\},$$

where $Q_a^b, \bar{u}_\rho^b, l_\sigma^\rho$ and $\bar{v}_a^\sigma$ respectively are the components of matrices $(Q), \bar{V}, O$ and $\bar{V}^{-1}$.

Hence, we have

$$tr(Q) = m - tr(B) + \sum_{A=m+1}^{n} \lambda_B.$$

Proposition 3.4 *Let N be an n-dimension sub-manifold and m be a co-dimension in a divine Kaehlerian manifold* $(\widetilde{N}, \widetilde{h}, \widetilde{Q})$ *and let* $(Q, h, V_a, \mu_a, (e_{ab})_n)$ *be the induced structure in N by* $(\widetilde{h}, \widetilde{Q})$*, then N is an inspired structure* $(\widetilde{h}, \widetilde{Q})$ *which is invariant on N is a divine Kaehlerian structure, whenever Q is non-trivial.*

Proof: If N is an invariant sub-manifold in a divine Kaehlerian structure $(\widetilde{N}, \widetilde{h}, \widetilde{Q})$ the $(\widetilde{Q}, \widetilde{h})$ is a divine Kaehlerian manifold

Conversely, if we assume that $(\widetilde{N}, \widetilde{h}, \widetilde{Q})$ is a divine Kaehlerian structure then

$$\sum_a (u_a(Y))^2 = 0$$

and we get

$$\sum_a u_a(Y) h(Y, u_a) = \sum_a (u_a(Y))^2 = 0,$$

where $u_a(Y) = 0$ for $a \in (1,2,..,m)$ and hence N is invariant.

Example: -

We see that the vast space is a $(m+n)$ –dimensional Euclidian space F^{m+n} $(m, n \in M^*)$.

Let $\tilde{Q}: F^{m+n} \to F^{m+n}$ be a $1,1$ –tensor field prescribed by

$$\tilde{Q}(x^1, \dots, x^m, y^1, \dots, y^n)$$
$$= (\Psi x^1, \dots, \Psi x^m, (\Psi - 1)y^1, \dots, (\Psi - 1)y^n)$$

For each point

$(x^1, \dots, x^m, y^1, \dots, y^n) \in F^{m+n}$ $where$ $\Psi = \frac{1+\sqrt{5}}{2}$ and $1 - \Psi = \frac{1-\sqrt{5}}{2}$

are the real roots of the equation $y^2 = y + 1$.

On the other side

for $(x^1, \dots, x^m, y^1, \dots, y^n), (u^1, \dots, u^m, v^1, \dots, v^n) \in F^{m+n},$

we have

$$\tilde{Q}^2(x^1, \dots, x^m, y^1, \dots, y^n) = (\Psi^2 x^1, \dots, \Psi^2 x^m, (1-\Psi)^2 y^1, \dots, (1-\Psi)^2 y^2)$$
$$= (\Psi x^1, \dots, \Psi x^m, (1-\Psi)y^1, \dots, (1-\Psi)y^n + (x^1, \dots, x^m, y^1, \dots, y^n)$$

Thus, we get $\tilde{Q}^2 = \tilde{Q} + I$

and

$$< \tilde{Q}(x^1, \dots, x^m, y^1, \dots, y^n), (u^1, \dots, u^m, v^1, \dots, v^n) >$$
$$= < (x^1, \dots, x^m, y^1, \dots, y^n), \tilde{Q}(u^1, \dots, u^m, v^1, \dots, v^n) >$$

for $every$ $point$ $(x^1, \dots, x^m, y^1, \dots, y^n), (u^1, \dots, u^m, v^1, \dots, v^n) \in F^{m+n}$

Therefore, the scalar product on F^{m+n} is relevant to $\tilde{Q}$ is the divine structure on $F^{m+n}, <>$ and $(F^{m+n}, <>, \tilde{Q})$ and hence it represents the divine Kaehlerian manifold.

Conclusion

Here is the brief discussion over some crucial results of this article:
Section 1 is introductory one, that includes the basic concepts related to divine Kaehlerian manifold which is Riemann manifold of even dimension, and whose holonomy group is contained in the unitary group.
In section 2, the property of divine Kaehlerian manifold in terms of Fibonacci sequence, trace and Eigen values of the divine structure in a Kaehlerian manifold has been studied. Furthermore, it is investigated that each 'almost product structure' in an n- dimensional Kaehlerian manifold induced the two divine structures.
In section 3, properties of the structure induce other structures and linearly independent existence for n-dimensional sub-manifolds in a Kaehlerian manifolds were studied. The inspired structured in a divine Kaehlerian manifold and induced structure in terms of non-trivial invariant have also been investigated.

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