J. Mountain Res. P-ISSN: 0974-3030, E-ISSN: 2582-5011
Vol. 16(1), (2021), 209-213 DOI: https://doi.org/10.51220/jmr.v16i1.21

Generating Vector Fields of the Metric Semi-Symmetric Connection on Almost Hyperbolic Kaehlerian Manifolds

Umraw Singh Negi[1*] • Preeti Chauhan[1]

[1]*Department of Mathematics, H.N.B. Garhwal University (A Central University), S.R.T. Campus Badshahi Thaul, Tehri Garhwal – 249 199, Uttarakhand, India*

Corresponding Author Email: usnegi7@gmail.com

Received: 17.4.2021; Revised: 11.5.2021; Accepted: 14.5.2021

Abstract: Negi and Semwal (2011), have studied on almost Kaehlerian conformal recurrent and symmetric Manifolds. In this paper, we have defined and calculated Generating vector fields of the metric semi-symmetric connection (**MS-Sc**) on almost Hyperbolic Kaehlerian Manifolds and its some theorems established**.**

Keywords: Geodesic Line • Induced and Isotropous Vector Fields • Hyperbolic Kaehlerian Manifolds

Mathematics subject classification 2010: 53A30, 32C15, 53C55.

1. Introduction:

A $n\,(=2m)$ dimensional Riemannian space M, with metric tensor (g_{ij}) indicates the structure by $\left\{ {i \atop j\ k} \right\}$ of covariant differentiation in the direction of the Levi-Civita connection **(L-Cc)** by $\dot{\nabla}$ and modules of its curvature tensor by K^i_{jkl} (or K_{ijkl}). The geodesic line of **L-Cc** is differentiated by the following [Mizusawa and Koto (1960)]:

(1.1) $v^k \dot{\nabla}_k v^j = 0,$

where v^k locates a module of tangent vector field of the geodesic line.

The modules of **MS-Sc** are known as:

(1.2) $\Gamma^a_{ik} = \left\{ {a \atop i\ k} \right\} + p_i\,\delta^a_k - p^a\,g_{ik}$

where p_i and p^a are known as producer or induced vector field of **MS-Sc**. The torsion tensor is equal to [Goldberg (1956)]:

(1.3) $T^a_{ik} = p_i\delta^a_k - p^k\delta^a_i$

if (1.3) holds for v^k, then:

(1.4) $p_k\, v^k v^j \;=\; p^j v_k\, v^k$

while, we reflect on a Riemannian space and its metrics is absolutely exact then (1.3) reduces:

(1.5) $p_k \;=\; \alpha\, p_k$

where α is a scalar function.

But here effect of the function α is the comparatively concurrent involving the geodesic line and induce vector field of **MS-Sc**. Allocate us indicate the machinist of covariant differentiation in the direction by ∇, then

$$\nabla_k v_j = \dot{\nabla}_k v_j - \alpha(v_k v_j - g_{kj})v$$

$$\dot{\nabla}_k p_j = \alpha_k v_j + \alpha \dot{\nabla}_k v_j,$$

where v sets used for scalar four-sided figure of the vector v_k with $\alpha_k = \partial\alpha / \partial x^k$.

Now modules of Riemannian curvature tensor of **MS-Sc** can be communicated herein mode:

(1.6) $$R_{ijkl} = K_{ijkl} + g_{ik} p_{lj} - g_{il} p_{kj} + g_{jl} p_{ki} - g_{jk} p_{li}$$

where K_{ijkl} signify Riemann curvature tensor module of **L-Cc** and contraction p_{kj} locates for:

(1.7) $$p_{kj} = \dot{\nabla}_k p_j - p_j p_k + \frac{1}{2} p_s p^s g_{jk}.$$

The tensor p_{kj} is symmetric if and only if p_j is a gradient, that means

$$\alpha_k v_j - \alpha_j v_k = \alpha(\dot{\nabla}_j v_k - \dot{\nabla}_k v_j),$$

Or, equivalently

(1.8) $$\frac{\partial v}{\partial x^k} = \frac{2}{\alpha} (\mu v_k - v\alpha_k)$$

where μ stands for $\alpha_k v^k$.

Since the appearance (1.5), we simply obtain [Yano (1965)]:

$$R_{jk} = K_{jk} + (2-n)p_{kj} - g_{jk}p_s^s,$$

$$g_{kj}p_s^s = K_{jk} - R_{jk} - (n-2)p_{jk},$$

$$np_s^s = K - R - (n-2)p_s^s,$$

$$p_s^s = (K-R) / 2(n-1),$$

$$p_{kj} = \frac{K_{jk} - R_{jk}}{n-2} - \frac{K-R}{2(n-1)(n-2)} g_{jk}.$$

Here K_{jk}, K and R_{jk}, R, indicate Ricci tensor, curvature scalar for the Levi-Civita connection respectively.

Here, comprise the curvature tensor equation (1.5) convince the entire statistical conditions which are mainly universal for Riemannian curvature tensors to exist skew-symmetric into primary both indices regular in modify sets of primary with next couple of indices and to convince of initial Bianchi identity. The entire properties be fulfilled iff the induced vector field is an inclined [Prvanovic and Pusic (1995)].

There holds:

$$v^i \dot{\nabla}_i v_k = 0 \quad \text{and} \quad p^i \dot{\nabla}_i p_k = p^i \dot{\nabla}_k p_i = \varphi p_k = \varphi \alpha v_k.$$

Next, concern the Ricci character for connection to the producer and then we find:

(1.9) $$v\alpha_j - \varphi v_j = 0.$$

Then there yields, in view of (1.7), $\frac{\partial v}{\partial x^k} = 0,$ and from (1.8), we have

$$\alpha_k = \frac{\varphi}{v} v_k, \quad \text{or} \quad \alpha_k = f\, p_k.$$

This represents every three vectors are mutually comparative. Therefore:

$$p_s p^s = \alpha^2 v$$

Besides:

$$\frac{\partial(p_s p^s)}{\partial x^k} = p^s \dot{\nabla}_k p_s + p_s \dot{\nabla}_k p^s = p^s \alpha_k p_s + \alpha p^s \dot{\nabla}_k v_s$$
$$= 2\alpha_k p_s p^s + 2\alpha p^s \dot{\nabla}_k v_s = 2\alpha_k p_s p^s + 2\alpha^2 v^s \dot{\nabla}_k v_s$$
$$= 2 p_s p^s \alpha_k = 2\alpha^2 \alpha_k v.$$

As $\dot{\nabla}_k v^s v_s = 0$ and consequently $v^s \dot{\nabla}_k v_s = 0,$
Then, on the other side

$$\frac{\partial(p_s p^s)}{\partial x^k} = 2 p^s \dot{\nabla}_k p_s = 2\alpha_k p^s p_s = 2\alpha^2 v \alpha_k$$
$$= 2\alpha v\, \alpha_k.$$

Equating above effects for $\frac{\partial(p_s p^s)}{\partial x^k}$, we find $\alpha = 1$ or $\alpha_k = 0$. Then:

(1) (p_k) and (v_k) are equivalent, both slopes, both of regular span.

or

(2) (p_k) and (v_k) are smooth vectors, equally of regular span and equally slopes.

Definition (1.1): In a Riemannian manifold, the curvature tensor of **MS-Sc** fulfilled entire extremely regular statistical conditions of the **L-Cc** and tangent vector fields of the geodesic lines are smooth inclines of constant length.

2. Generating Vector Fields of the Metric Semi-Symmetric Connection on Almost Hyperbolic Kaehlerian Manifolds:

An almost hyperbolic Kaehlerian manifolds are 2n-dimensional pseudo-Riemannian manifolds, capable by a non-disintegrate formation tensor F_j^i gratifying:

(2.1) $$F_j^i F_k^j = \delta_k^i, \qquad F_{ij} = -F_{ji}, \qquad \dot{\nabla} F_{ij} = 0.$$

Here, we know that a almost hyperbolic Kaehlerian manifolds is in fact a product manifold, but its covariant structure tensor is skew-symmetric. Besides, the structure tensor has n linearly independent Eigen vectors, by the fact of skew-symmetry of the structure, it sends any vector into an orthogonal vector and Eigen vectors of the structure are consequently, self-orthogonal. In any point of a hyperbolic almost Kaehlerian manifold, its tangent space can be spanned by these self-orthogonal vectors, it is its adapted basis. It is obvious that these are two Eigen subspaces of equal dimension, for two structure's eigenvalues, **1 and -1.** on both these invariant subspace, the metric tensor vanishes. Actually, a hyperbolic almost Kaehlerian manifolds in any point is a product of two totally geodesic subspaces. This is the reason to investigate geodesic lines of this kind of space. Here we may have such geodesic lines which are minimizing the distance between two different points up to zero, from one point, we may reach another point instantly along such a geodesic line.

Theorem (2.1): Proved that Riemannian spaces curvature scalar of Levi-Civita connection of **MS-Sc** and F-connection are commonly equivalent.

Proof: We have **MS-Sc** of an almost hyperbolic Kaehlerian manifolds contain the torsion tensor:

(2.2) $$T_{ij}^k = p_i \delta_j^k - p_j \delta_i^k + q_i K_j^k - q_j F_i^k,$$

where p_i and q_i are modules of positive vector fields. If this connection to be a metric one, then its modules:

(2.3) $H^a_{ik} = \left\{ {a \atop i \;\; k} \right\} + p_i \delta^a_k - p^a g_{ik} - q_k F^a_i$

If this signify that $\nabla F = 0$, then

(2.4) $q_j = -\frac{n}{2} p_a F^a_j$, $p_a F^a_j = -\frac{2}{n} q_j$

Then we can denote:

(2.5) $H^a_{ik} = \Gamma^a_{ik} - q_k F^a_i$

where Γ^a_{ik} exists module of Riemannian spaces and module of identical on the adjunct pseudo-Riemannian manifolds, fulfilling forms of definition (1.1) [Gray (1967)].

At present, compute the coefficients of (2.3), we get:

$$\bar{R}_{ijkl} = R_{ijkl} - F_{ji}\left(\dot{\nabla}_l q_k - \dot{\nabla}_k q_l\right) + q_k\left(p_j F_{li} + \frac{2}{n} q_j g_{li} + \frac{2}{n} q_i g_{lj} + p_i F_{jl}\right) - q_l\left(p_j F_{ki} + \frac{2}{n} q_j g_{ki} + \frac{2}{n} q_i g_{kj}\right) + p_i F_{jk}$$

By R_{ijkl} indicate a module of curvature tensor gratifying the def (1.1) and $\bar{R}_{ijkl}$ is skew-symmetric into initial indices. $\bar{R}_{ijkl}$ is consistent in growing places of primary and next couple of indices iff the tensor ($p_l q_k + q_l p_k$) is skew-symmetric. Then:

$$p_k p^k q_l = -p^k q_k p_l.$$

As the vectors p^k and q^k are mutually orthogonal, there yields $p_k p^k = 0$. Its indicates the producer on the Riemannian spaces of F-connection is an isotropous slope and harmony through declaration of definition (1.1). Therefore vector q_k as well isotropous slope.

Theorem (2.2): The curvature tensor of a **MS-Sc** and F-connection on the almost hyperbolic Kaehlerian manifolds invariable in varying sets of primary and next couple of index and making the primary Bianchi identity iff the producers on association are isotropous i.e. $\dot{\nabla}_a p^a = 0$ and conversely.

Proof: We have from (2.3), the pseudo-Riemannian manifolds structures and v^i is satisfied following:

(2.6) $p_j \delta^i_k v^j v^k - p^i q_j v^j v^k - q_k F^i_j v^j v^k = \frac{2}{n} p_j v^j v^i$

(2.7) $(p_j v^j - v_j v^j - \frac{2}{n} p_j v^j) v^i = - q_k v^k u^i,$

where $u^i = - F^{ji} v_j = F^{ij} v_j.$

Then, from (2.7),

(2.8) $u^i = \alpha v^i$, or $q_k v^k = 0.$

If the eigen values **1 or -1,** therefore $q_k v^k$ are equals to:

$$\frac{n-2}{n} p_j v^j - v_j v^j = \frac{n-2}{n} p_j v^j.$$

and from (2.8), correspond the vectors modified:

(2.9) $p = p^a l_a + p^{\hat{b}} l_{\hat{b}}$,

where $l_{\hat{b}}$ are eigen vectors, used for eigen values **-1,** we get:

$$q = -\frac{2}{n} p^a l_a + \frac{2}{n} p^{\hat{b}} l_{\hat{b}} \quad \text{and} \quad v = v^a l_a + v^{\hat{b}} l_{\hat{b}}.$$

Then (2.8) gives:

(2.10) $$q_k v^k = \frac{2}{n}\left(p^{\hat{b}} v^a - p^a v^{\hat{b}}\right) g_{a\hat{b}} = 0,$$

That is fulfilled; as well as v is comparative to p and isotropous.

While the almost hyperbolic Kaehlerian manifolds induced vector fields of similar line containing several its autoparallel lines in ordinary with **L-Cc** is isotropous, (1.6) appears that procedure:

(2.11) $$p_{kj} = \dot{\nabla}_k p_j - p_k p_j$$

and

(2.12) $$\dot{\nabla}_s p^s = p^s_s = \frac{K-R}{2(n-1)}.$$

Constricting the tensor ($\dot{\nabla}_k q_l - \dot{\nabla}_l q_k$) through the tensor F^l_b, we attain

$$-\frac{2}{n}\dot{\nabla}_k p_b + \frac{2}{n} F^l_b F^a_k \dot{\nabla}_l p_a = -\frac{n}{2}\dot{\nabla}_k p_b + F^l_b \dot{\nabla}_l q_k$$

and
$$\frac{n-4}{2n}\dot{\nabla}_k p_b = F^l_b\left(\dot{\nabla}_l q_k - \frac{2}{n} F^a_k \dot{\nabla}_l p_a\right).$$

Contracting the last relation with g^{kb}, we obtain:

$$\frac{n-4}{2n}\dot{\nabla}_a p^a = -\frac{2}{n}\dot{\nabla}_a p^a + \frac{2}{n} p_a p^a = 0.$$

If $n > 4$, then

(2.13) $$\dot{\nabla}_a p^a = 0.$$

Then, by (2.12), $K = R.$

On the other hand, from (2.5) and the shape of the curvature tensor, we can obtain that:

(2.14) $$\overline{R} = R + F^{lk}\left(\dot{\nabla}_l q_k - \dot{\nabla}_k q_l\right)$$

or
$$\overline{R} = R + \frac{2}{n}\left(\dot{\nabla}_l p^l + \dot{\nabla}_l p^l\right) = R + \frac{4}{n}\dot{\nabla}_l p^l.$$

Therefore this tensor to gratify initial Bianchi identity with appearance for warp tensor, after that established:

(2.15) $$\dot{\nabla}_a p^a = 0.$$

Hence theorem (2.2) is proved.

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