ON HYPER DABOUX LINES IN KAHLERIAN SPACES

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Received- 26-03-09 Accepted-30-12-09

ABSTRACT

A darboux line (or D-line) on a surface of 3-dimensional Euclidean space is a curve for which the osculating sphere at each point is tangent to the surface. Some properties of these curves have been studied by Semin ([4], [5]). These curves have been generalized to give the hyper Darboux lines (or hyper D-lines) of a surface (Prvanovitch [3]).

In the present paper, we have defined and studied the hyper Darboux lines of order h (h = 0, 1, 2 ..., n-3) for a K_{n} imbeded in a Kaehlerian space K'_{n}. Some properties of Union hyper D-lines have also been investigated.

Keywords:- Kaehlerian Space, Reimannian curvature tensor, Darboux lines, Hyper Darboux lines of hth order, Union hyper D-line, Zero torsion (second curvature), Union curve.

INTRODUCTION

An even dimensional Kaehlerian space is a Riemannian space, which admits a structure tensor field F^{h} satisfying the relations (Yano, 1965[7]) :

(1.1) \quad F_{j}^{i} F_{i}^{h} = -\delta_{j}^{h},

(1.2) \quad F_{ij} = - F_{ji}, \quad \left( F_{ij} \overset{\text{def}}{=} F_{i}^{j} g_{nj} \right)

and

(1.3) \quad F_{i,j}^{h} = 0 ,

where the comma (,) followed by an index denotes the operator of covariant differentiation with respect to the metric tensor g_{ij} of the Riemannian space.

The Riemannian Curvature Tensor, R^{h}_{i,jk} is given by

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\[ R_{ij} = \partial_i \left\{ \begin{array}{c} h \\ j \\ k \end{array} \right\} - \partial_j \left\{ \begin{array}{c} h \\ i \\ k \end{array} \right\} + \left\{ \begin{array}{c} h \\ a \\ j \\ k \end{array} \right\} - \left\{ \begin{array}{c} h \\ j \\ a \\ i \end{array} \right\} \]

where as the Ricci tensor and the scalar curvature are given by \( R_{ij} = R^a_{\ aij} \) and \( R = R_{ij} g^{ij} \) respectively.

It is well known, that these tensors satisfy the identities, (Tachibana, 1967 [6]):

\[ (1.4) \quad F^a_i R^j_a = R^a_i F^j_a \]

and

\[ (1.4a) \quad F^a_i R^j_a = -R^a_i F^j_a. \]

In view of (1.1), the relation (1.4) gives

\[ (1.5) \quad F^a_i R^b_a F^j_b = -R^j_i. \]

Also, multiplying (1.4a) by \( g^{ij} \), we get

\[ F^a_i R^j_a = -R^j_i F^a_a, \]

which implies

\[ (1.6) \quad F^a_i R^j_a = 0. \]

If we define a tensor \( S_{ij} \) by

\[ (1.7) \quad S_{ij} = F^a_i R^j_a, \]

we have

\[ (1.8) \quad S_{ij} = -S_{ji}. \]

Hyper D-lines of order \( h \):

Let an \( m \)-dimensional Kaehlerian subspace \( \kappa_m \) given by the equation

\[ y^a = y^a (x^i), \quad (\alpha = 1, 2, \ldots, n; i = 1, 2, \ldots, m) \]

be immersed in a Kaehlerian space \( \kappa_n \).

Let \( C : x^i = x^i (s) \) be a curve (not a geodesic of the enveloping space) of the subspace \( \kappa_m \).

The components \( \eta^a_i \left( = \frac{dy^a}{ds} \right) \) of the unit tangent vector, \( \eta^a_i \) of the principal normal and \( \eta^a_i \) (\( r = 2, \ldots, m-1 \)) of the \( r \)-th binormal vector define an orthogonal system of unit vectors at every point of the curve. Assuming that \( \frac{\delta}{\delta s} \) in the usual covaricint
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In particular, if the vector \( R_{(1)}^{a} \eta_{(1)}^{a} + R_{(2)}^{a} \left( \frac{dR_{(1)}}{ds} \right) \eta_{(2)}^{a} \) is along the normal \( N^{a} \), the D-line is given by

\[
3\Omega_{(v)}(p) \left( \frac{dx^{i}}{ds} \right) + \Omega_{(v),k} \left( \frac{dx^{i}}{ds} \right) \left( \frac{dx^{j}}{ds} \right) \left( \frac{dx^{k}}{ds} \right) + \\
\sum_{\mu} \Omega_{(\mu),i} \left( \frac{dx^{i}}{ds} \right) \left( \frac{dx^{j}}{ds} \right) \theta_{1,\mu,ij} \left( \frac{dx^{k}}{ds} \right) = 0.
\]

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