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ON HYPER DABOUX LINES IN KAEHLERIAN SPACES

A.K. SINGH AND A.K. GAHLOT
Department of Mathematics, H.N.B. Garhwal University Campus,
Badshahi Thaul, Tehri Garhwal - 249199, Uttarakhand (India)

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ABSTRACT

A darboux line (or D-line) on a surface of 3-dimensional Euclidean space is a curve for which the osculating sphere at each point is tangent to the surface. Some properties of these curves have been studied by Semin ([4], [5]). These curves have been generalized to give the hyper Darboux lines (or hyper D-lines) of a surface (Prvanovitch [3]).

In the present paper, we have defined and studied the hyper Darboux lines of order h(h = 0, 1, 2 ...., n-3) for a K_m imbeded in a Kaehlerian space K_n . Some properties of Union hyper D-lines have also been investigated.

Keywords:- Kaehlerian Space, Reimannian curvature tensor, Darboux lines, Hyper Darboux lines of hth order, Union hyper D-line, Zero torsion (second curvature), Union curve.

INTRODUCTION

An even dimensional Kaehlerian space is a Riemannian space, which admits a structure tensor field F_i^h satisfying the relations (Yano, 1965[7]) :

(1.1).... $F_j^i \; F_i^h = -\delta_j^h,$

(1.2).... $F_{ij} = -F_{ji},$ $\left(F_{ij} \overset{def}{=} F_i^a g_{aj}\right)$

and

(1.3).... $F^h_{i,j} = 0,$

where the comma (,) followed by an index denotes the operator of covariant differentiation with respect to the metric tensor g_{ij} of the Rieamannian space.

The Riemannian Curvature Tensor, R^h_{ijk} is given by

$$R^{h}_{ijk} = \partial_i \begin{Bmatrix} h \\ j\ k \end{Bmatrix} - \partial_j \begin{Bmatrix} h \\ i\ k \end{Bmatrix} + \begin{Bmatrix} h \\ i\ a \end{Bmatrix} \begin{Bmatrix} a \\ j\ k \end{Bmatrix} - \begin{Bmatrix} h \\ j\ a \end{Bmatrix} \begin{Bmatrix} a \\ i\ k \end{Bmatrix},$$

where as the Ricci tensor and the scalar curvature are given by $R_{ij} = R^{a}_{aij}$ and $R = R_{ij} g^{ij}$ respectively.

It is well known, that these tensors satisfy the identities, (Tachibana, 1967 [6]) :

(1.4).... $F^{a}_{i} R^{j}_{a} = R^{a}_{i} F^{j}_{a}$

and

(1.4a).... $F^{a}_{i} R_{aj} = - R_{ia} F^{a}_{j}$.

In view of (1.1), the relation (1.4) gives

(1.5).... $F^{a}_{i} R^{b}_{a} F^{j}_{b} = - R^{j}_{i}$.

Also, multiplying (1.4a) by g^{ij}, we get

$$F^{a}_{i} R^{i}_{a} = - R^{j}_{a} F^{a}_{j},$$

which implies

(1.6).... $F^{a}_{i} R^{i}_{a} = 0$.

If we define a tensor S_{ij} by

(1.7).... $S_{ij} = F^{a}_{i} R_{aj}$,

we have

(1.8).... $S_{ij} = - S_{ji}$.

Hyper D-lines of order h:

Let an m–dimensional Kachlerian subspace K_m given by the equation $y^{\alpha} = y^{\alpha}(x^{i})$, $(\alpha = 1, 2, ... n\,; i = 1, 2, ... m)$ be immersed in a Kaehlerian space K_n.

Let $C : x^{i} = x^{i}(s)$ be a curve (not a geodesic of the enveloping space) of the subspace K_m.

The components $\eta^{\alpha}_{(o)} \left(\equiv \frac{dy^{\alpha}}{ds} \right)$ of the unit tangent vector, $\eta^{\alpha}_{(1)}$ of the principal normal and $\eta^{\alpha}_{(r)}$ (r = 2, . . ., m –1) of the (r – 1)th binormal vector define an orthogonal system of unit vectors at every point of the curve. Assuming that $\delta/\delta s$ in the usual covaricint

In particular, if the vector $R_{(1)}\eta^{\alpha}_{(1)} + R_{(2)}\left(\frac{dR_1}{ds}\right)\eta^{\alpha}_{(2)}$ is along the normal N^{α}_{ν}, the D-line is given by

$$3\Omega_{(\nu)ij}p^{i}\left(\frac{dx^{j}}{ds}\right) + \Omega_{(\nu)ij.k}\left(\frac{dx^{i}}{ds}\right)\left(\frac{dx^{j}}{ds}\right)\left(\frac{dx^{k}}{ds}\right) +$$

$$+\sum_{\mu}\Omega_{(\mu)ij}\left(\frac{dx^{i}}{ds}\right)\left(\frac{dx^{j}}{ds}\right)\theta_{(\nu\mu)k}\left(\frac{dx^{k}}{ds}\right) = 0 .$$

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