RELIABILITY ANALYSIS A THREE UNIT STANDBY SYSTEM

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ABSTRACT

This paper presents the stochastic model of a three unit redundant system having two types of failures. Two different approaches have been applied to the system. First the numerical solution of the model and the second Laplace Transform Technique has been applied. Euler’s modified method has been used to solve the differential-difference equations representing the system to obtain all state probabilities and the availability of the system. Similarly Laplace Transform technique has also been applied to this model. The results obtained by these techniques have been compared.

Keywords-Reliability, Availability, Euler’s modified method, MTTF and Laplace Transform.

INTRODUCTION

Reliability modeling of standby systems requires involvement of several states to represent a true picture of the functioning of the system. Further in the mathematical modeling of such systems, one gets system of differential-difference equations with the number of equations equaling the number of involved states. With large number of states and hence equations, the Laplace Transform (LT) technique for solving system of differential equations becomes tedious. Under the circumstances researchers usually make a compromise at the modeling stage by sacrificing some significant constraints to reduce the number of states and avoid complexities of LT technique with large number of equations. Sur & Sarkar (1996) used a numerical method to solve the system of differential-difference equations and demonstrated that the approximate results obtained by numerical method match considerably with the results obtained by LT technique. Present work studies the stochastic behaviour of a three unit standby system having two standby units and two types of failure modes. Using Euler’s modified method to solve the system of differential-difference equations representing the system, we have obtained the system state probabilities for success and failure states along with the computation of availability of the system.
The system state transition diagram is given in fig. 1. System under consideration consists of three units out of which one operates and the other two works as standbys, one warm and the other cold standby. State $S_0$ represents the state where all units are in perfect condition. If the operative unit suffers a complete failure or the warm standby fails in standby mode, it transits to state $S_1$, where the other unit takes over as operative unit and failed unit goes for repair. If the operative unit suffers a partial failure, then it transits to state $S_2$. Upon the complete failure of operative unit or failure of warm standby in state $S_1$, system arrives in completely failed state $S_5$. If in state $S_1$ the operative unit fails partially, the system transits to state $S_3$. This state is also arrived from state $S_2$ with the complete failure of operative unit. However the partial failure of the operative unit in state $S_2$ leads to state $S_4$. A partially failed unit may fail completely and under this situation system transits from $S_4$ to $S_3$ and $S_3$ to $S_5$. From states $S_3$ and $S_4$, the system is led to completely failed state $S_5$, if the operative unit fails in either mode. The system is assumed to be repairable with repairs taken to be as good as new.

![System State Transition Diagram](image)

**Assumptions**
1. An operative unit may fail completely or partially.
2. At least two units in working condition must be available to the system for its functioning.
3. Partially failed unit may work as warm standby.
4. Failure of warm standby in standby mode is always a complete failure.
5. A partially failed unit is repaired in priority to completely failed unit.

**Notations**

- $P_i(t) = \text{Probability that at any time } t \text{ the system is in state } i; [i = 0, 1, 3, 4]$.
- $P_j(x, t) = \text{Probability density function (with respect repair time) that the Failed system is in state } j \text{ and has an elapsed repair time } x$
  - $= \text{Constant complete failure rate.}$
  - $= \text{Complete failure rate in standby mode.}$
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1. presents these state probabilities along with system availability for a range of values for \( t = 0(10) 100 \) where availability is given by

\[
A(t) = \sum_{i=0}^{4} (t)
\]

Table-3

<table>
<thead>
<tr>
<th>Time</th>
<th>State 0</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
<th>State 5</th>
<th>( A(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.6448</td>
<td>0.1667</td>
<td>0.1471</td>
<td>0.0205</td>
<td>0.0100</td>
<td>0.0110</td>
<td>0.9889</td>
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<tr>
<td>20</td>
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<td>0.2172</td>
<td>0.1638</td>
<td>0.0684</td>
<td>0.0302</td>
<td>0.0669</td>
<td>0.9331</td>
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<td>30</td>
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<td>0.2208</td>
<td>0.1440</td>
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<td>0.1450</td>
<td>0.8549</td>
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<tr>
<td>40</td>
<td>0.2939</td>
<td>0.2079</td>
<td>0.1205</td>
<td>0.1079</td>
<td>0.0501</td>
<td>0.2195</td>
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<td>50</td>
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<td>0.1921</td>
<td>0.1028</td>
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<td>0.2776</td>
<td>0.7224</td>
</tr>
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<td>0.1785</td>
<td>0.0921</td>
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<td>0.0848</td>
<td>0.0362</td>
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<tr>
<td>100</td>
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<td>0.0855</td>
<td>0.0826</td>
<td>0.0353</td>
<td>0.3567</td>
<td>0.6432</td>
</tr>
</tbody>
</table>

System state probabilities & availability vs Time

CONCLUSION

It can be observed from the graph given in Fig-4 that initially, system oscillates in states \( S_0, S_1, \) and \( S_2 \) for quite some time. These graphs start decreasing after that and graphs for state \( S_3 \) and \( S_4 \) start steadily increasing. The graphs of the failed state \( S_5 \)
starts increasing after some time of start and continues to increase. Its increments are sharp in the beginning and becomes steady at a later stage. The study of the behaviour of the state probabilities of the system may be quite useful in system design. The overall availability curve decreases with increase in time.

REFERENCE


