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ON A SPECIAL TACHIBANA RECURRENT SPACE OF SECOND ORDER

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ABSTRACT

In the present paper, we have defined Special Tachibana recurrent space of second order and derived several interesting theorems there in.

KEY WORDS : Almost Tachibana Space, Tachibana Space, Reimannian matric, Special Tachibana recurrent space of second order

INTRODUCTION

Let us consider m (= 2n) dimentional Real mainfold M_{2n} of differentiability class (C^r) with respect to an allowable Co-ordinate system :

$$\left(x^i, x^{\bar{i}}\right) \equiv \left(x^1, x^2, \ldots\ldots, x^{n+1}, x^{\bar{1}}, x^{\bar{2}}, \ldots\ldots, x^{\overline{n+1}}\right)$$

If there exist a mixed tensor $F_i^h\left(x^i, x^{\bar{i}}\right)$ of class C^r, which satisfies

$$F_j^i F_i^h = -A_j^h, \qquad \ldots (1.1)$$

and the Riemannian matric g_{ij} satisfying :

$$dS^2 = g_{i\bar{j}}\left(x, \bar{x}\right) dx^i \, dx^{\bar{j}}, \qquad \ldots (1.2)$$

which also satisfies the condition,

$$\nabla_j F_{ih} + \nabla_i F_{jh} = 0, \qquad \ldots (1.3)$$

then, the space is called an almost Tachibana Space. If the conditions

$$\frac{\partial^2 x^h}{\partial x^j \partial x^i} + \frac{\partial x^k}{\partial x^i} g^{h\bar{s}} \partial_k g_{j\bar{s}} - \frac{\partial x^h}{\partial x^k} g^{k\bar{s}} \partial_i g_{j\bar{s}} = 0 \qquad \ldots (1.4)$$

and

$$\frac{\partial^2 x^{\bar{h}}}{\partial x^{\bar{j}} \partial x^{\bar{i}}} + \frac{\partial x^{\bar{k}}}{\partial x^{\bar{i}}} g^{\bar{h}s} \partial_{\bar{k}} g_{\bar{j}s} - \frac{\partial x^{\bar{h}}}{\partial x^{\bar{k}}} g^{\bar{k}s} \partial_{\bar{i}} g_{\bar{j}s} = 0 \qquad \ldots (1.5)$$

are satisfied, then the space is said to be a Tachibana space.

Let us, now, consider an affinely connected n-dimensional Tachibana recurrent space of second order, whose curvature tensor R^h_{ijk} satisfies the following condition :

$$\nabla_n \nabla_m R^h_{ijk} = \lambda_{mn} R^h_{ijk}, \qquad \ldots (1.6)$$

where λ_{mn} is a non-symmetric, in general and non vanishing covariant tensor. We shall assume to put the following two conditions in our space :

$$\nabla_j \upsilon^i = \phi_j \upsilon^i \qquad \ldots (1.7) \text{ and}$$

$$R_{jk} = \phi_j \alpha_k, \qquad \ldots (1.8)$$

where α_k means a suitable covariant tensor, R_{kl}, we have

$$\nabla_n \nabla_m R_{ij} = \lambda_{mn} R_{ij} \qquad \ldots (1.9)$$

Making use of (1.8) in (1.9), we have

$$\nabla_n \alpha_j \nabla_m \phi_i + \alpha_j \nabla_n \nabla_m \phi_i + \nabla_n \phi_i \nabla_m \alpha_j + \phi_i \nabla_n \nabla_m \alpha_j = \alpha_{mn} \phi_i \alpha_j \qquad \ldots (1.10)$$

In fact, when the space under consideration admits an affine motion

$$\bar{x}^i = x^i + \upsilon^i \delta t.$$

Characterized by (1.7), we have a resolved form of Ricci-tensor of the form (1.8). Here, we assume the existence of recurrent covariant vector υ^i given by (1.7) and in addition, the resolvability of R_{ij}.

Making a commutator on the indices m and n in (1.10), we obtain

$$-\alpha_j \phi_a R^a_{imn} - \phi_i \alpha_a R^a_{jmn} = A_{mn} \phi_i \alpha_j \qquad \ldots (1.11)$$

where, $A_{mn} \overset{def}{=\!=} \lambda_{mn} - \lambda_{nm}$.

Multiplying (1.11) by υ^i and summing over i and making use of (1.10), we have

$$\phi(A_{mn}\alpha_j + \alpha_a R^a_{jmn} + \alpha_j \Omega_{mn}) = 0,$$

where, $\phi \overset{def}{=\!=} \phi_i \upsilon^i$.

In the present paper, we have to discuss the next two cases :

In this way, the existence of ϕ_j is examined and we have here a characteristic condition on $\nabla_j \upsilon^h$:

$$\nabla_k \nabla_j \upsilon^h = \rho_k \nabla_j \upsilon^h, \quad \rho_k = \frac{\nabla_k \phi}{\phi}.$$

On the other hand, in case (2.4a), from (1.8), we obtain $R_{jk} = 0$.
Summarizing the above all conditions, we have the following :

Theorem (2.1). In a n-dimensional Tachibana recurrence space of second order, admitting a contravariant vector υ^h, characterized by

$$\nabla_j \upsilon^i = \phi_j \upsilon^i$$

and having a disjointed Ricci tensor of the form $R_{jk} = \phi_j \alpha_k$, there exist a case of $\alpha_k \upsilon^m = 0$:
In this case, if $\alpha_m = 0$, then we have the vanishing of Ricci tensor R_{jk} and if $\alpha_m \neq 0$, we have

$$R_{kl} - R_{lk} = \nabla_l \phi_k - \nabla_k \phi_l .$$

The mixed tensor $\nabla_j \upsilon^i$ itself is a recurrent one characterized by

$$\nabla_l \nabla_k \upsilon^i = \rho_l \nabla_k \upsilon^i$$

for a definite gradient vector $\rho_l = \nabla_l \phi / \phi$.

Remark. The latter part of the above theorem does not contain a case of ϕ_j being a gradient vector.
Such a case is a showing but worthless one, in fact, if ϕ_j will be a gradient vector, we have

$$\alpha_l \phi_k - \alpha_k \phi_l = \nabla_k \phi_l - \nabla_l \phi_k = 0,$$

from which we have

$$\lambda \phi_k \upsilon^k = \alpha_k \upsilon^k = \alpha = 0,$$

this is $\lambda \phi = 0$ from which being $\phi \neq 0$, it follows that $\lambda = 0$.
Hence, we have $\alpha_k = 0$.
This is a contradiction, for we have $\nabla_l \phi_k + \phi_k \phi_l = 0$ at this moment and the case of ϕ_k being a gradient vector occurs.

3.THE CASE OF $\phi = 0$ **:** Let us consider the case of (1.12b). Then using the analogous method used in section 2, multiplying (2.2) by υ^j, we obtain the following formula :

$$\phi_a R^a_{jmn} = -(A_{mn} + \Omega_{mn})\phi_j \qquad \ldots (3.1)$$

Substituting (3.1) into the left hand side of (1.11), we get

$$\phi_j(\alpha_a R^a_{kmn} - \alpha_k \Omega_{mn}) = 0 .$$

Hence, we have here two cases to be discussed. They are

$$\phi_j = 0$$

$$\alpha_a R^a_{kmn} = \alpha_k \Omega_{mn} \qquad \ldots (3.2b)$$

The case of (3.2a) yields one of $\nabla_i \upsilon^h = 0$ and $R_{ij} = 0$.

The case of (3.2b) may be treated as follows :

We have, $\alpha_j \Omega_{mn} + \alpha_m \Omega_{nj} + \alpha_n \Omega_{jm} = 0$,

from which by contradiction of υ^j, we have, $\alpha\Omega_{mn} = 0$,

because of $\Omega_{jk} \upsilon^j = -R_{ak} \upsilon^a = -\phi_a \alpha_k \upsilon^a = -\phi\alpha_k$.

From (1.10), by contraction on the indices h and j, and $\Omega_{mn} = -\Omega_{nm}$ and $\phi = \phi_j \upsilon^j$.

In this way, we obtain $\Omega_{mn} = 0$, say $\nabla_n \phi_m = \nabla_m \phi_n$.

Thus, we can state here the following :

Theorem (3.1). When $\phi = 0$ *in our space, there exist two cases. One of them is a case* of satisfying $\nabla_j \upsilon^i = 0$ and $R_{jk} = 0$ and the other is a case of $\nabla_l \phi_k = \nabla_k \phi_l$.

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