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A FIXED POINT THEOREM FOR SYSTEM OF TRANSFORMATION SATISFYING A GENERAL CONTRACTIVE CONDITION OF INTEGRAL TYPE ON PRODUCT SPACES

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ABSRACT

In this paper we establish a fixed point theorem for a system of transformations satisfying a general contractive inequality of integral type on product spaces. Our results generalize the fixed point theorems of Rhoades (2003) and Branciari (2002).

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INTRODUCTION

In a recent paper Branciari (2002) obtained a fixed point result for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. Rhoades (2003) extend the result of Branciari [op.cit.] by proving two fixed point theorems involving more general contractive conditions and further Vijayaraju et.al. (2005) extend the results of Rhoades [op. cit.] for two maps by establishing a general principle. On the other hand Matkowski (1973),(1975) gave an important generalization of Banach contraction principle for a system of transformations on finite product of metric spaces. This result has been extended and generalized by Baillon-Singh (1993), Czerwik (1976), Gairola et.al. (1995),(1997), Matkowski-Singh (1996), Reddy-Subrahmanyam (1981) and Singh et.al. (1994),(1991),(1995), (1986). The purpose of this paper is to extend and unify the results of Rhoades (2003), Branciari (2002) and Matkowski (1975).

Throughout this paper we generally follow the notations of Matkowski ((1973), (1975))(see, also Singh-Gairola (1991) and Singh-Kulshrestha (1986).

Let a_{ik} be non-negative real numbers, $i, k = 1, ..., n$, and $(c_{ik}^{(0)})$ be a square matrix with $c_{ik}^{(0)} \in R$ (set of real numbers),$i, k = 1, ..., n$. Define the sequence of matrices $(c_{ik}^{(t)})$as follows:

$$c_{ik}^{(0)} = \begin{cases} a_{ik} & , i \neq k \\ 1 - a_{ik} & , i = k \end{cases} i, k = 1, ..., n; \tag{1.1}$$

$$c_{ik}^{(t+1)} = \begin{cases} c_{11}^{t} c_{i+1,k+1}^{t} + c_{i+1,1}^{t} c_{1,k+1}^{t} & , i \neq k \\ c_{11}^{t} c_{i+1,k+1}^{t} - c_{i+1,1}^{t} c_{1,k+1}^{t} & , i = k \end{cases}, \tag{1.2}$$

$$t = 0, 1, ..., n-2; i, k = 1, ..., n-t-1.$$

Evidently,$(c_{ik}^{(t)})$ is an $(n-t) \times (n-t)$ square matrices.

The following lemma is essentially due to Matkowski (1975) (see, also Czerwik (1976) and Singh-Kulshrestha (1986)).

LEMMA1.1. Let $(c_{ik}^{(0)}) > 0$ for $i, k = 1, ..., n$, $n \geq 2$, then the system of inequalities

$$\sum_{k=1}^{n} a_{ik} r_k < r_i, i = 1, ..., n.$$

has a positive solutions $r_1, r_2, ..., r_n$ if and only if the following inequalities hold

$$c_{ii}^{(t)} > 0, t = 0, 1, ..., n-1; \quad i = 1, ..., n-t. \tag{1.3}$$

Let (X_i, d_i), $i = 1, ..., n$ be complete metric spaces.

$$X := X_1 \times X_2 \times ... \times X_n;$$

$$x := (x_1, ..., x_n) \ , \ x \in X;$$

$$x^m := (x_i^m, ..., x_n^m) \ , m \in N = \{1, 2, ...\}.$$

2. RESULT

Now we state our main result.

THEOREM 2.1. Let (X_i, d_i) be complete metric spaces and $S_i : X \to X_i, i = 1, ..., n$. If there exists non-negative real numbers $b < 1$ and a_{ik}, $i, k = 1, ..., n$ such that

$$\int_0^{d_i(S_i x, S_i y)} \varphi(t) dt \leq \max \left\{ \sum_{k=1}^{n} a_{ik} \int_0^{d_k(x_k, y_k)} \varphi(t) dt, b \int_0^{m_i(x,y)} \varphi(t) dt \right\} \tag{2.1}$$

fixed point theorem for a system of transformation

$$\int_0^{d_i(S_ix^n,S_ix^m)} \varphi(t)dt \leq \sum_{j=n}^{m-1} \int_0^{d_i(x_i^j,x_i^{j+1})} \varphi(t)dt. \tag{2.8}$$

Using (2.5) and (2.8) ,

$$\int_0^{d_i(S_ix^n,S_ix^m)} \varphi(t)dt \leq \sum_{j=n}^{\infty} \lambda^j r_i. \tag{2.9}$$

Taking the limit of (2.9) as $m,n \to \infty$, it follows that $\{x_i^m\}$ is a Cauchy sequence, hence convergent, since X_i is complete. Call the limit u_i. From (2.1)

$$\int_0^{d_i(S_iu,S_ix^m)} \varphi(t)dt \leq \max\left\{\sum_{k=1}^{n} a_{ik} \int_0^{d_k(u_k,x_k^m)} \varphi(t)dt, b\int_0^{m_i(u_i,x_i^m)} \varphi(t)dt\right\}$$

$$m_i(u_i,x_i^m) = \max\left\{d_i(u_i,S_iu), d_i(x_i^m,S_ix^m), \left[d_i(x_i^m,S_iu)+d_i(S_ix^m,u_i)\right]/2\right\}.$$

Letting $m \to \infty$

$$\int_0^{d_i(S_iu_i,u_i)} \varphi(t)dt \leq b\int_0^{d_i(S_iu_i,u_i)} \varphi(t)dt,$$

which is a contradiction. Hence $S_iu = u_i$, $i = 1,...,n$. Now we shall prove the uniqueness of u_i. Let there exists some $w_i \in X_i$ and $S_iw = w_i$; $i = 1,...,n$. Then using (2.1)

$$\begin{aligned}\int_0^{d_i(u_i,w_i)} \varphi(t)dt &\leq b\int_0^{d_i(S_iu,S_iw)} \varphi(t)dt \\ &\leq \max\left\{\sum_{k=1}^{n} a_{ik}\int_0^{d_k(u_k,w_k)} \varphi(t)dt, b\int_0^{m_i(u_i,w_i)} \varphi(t)dt\right\} \\ &\leq \max\left\{\sum_{k=1}^{n} a_{ik}\int_0^{d_k(u_k,w_k)} \varphi(t)dt, b\int_0^{d_i(u_i,w_i)} \varphi(t)dt\right\} \end{aligned} \tag{2.10}$$

Once again we can assume that

$$\int_0^{d_i(u_i,w_i)} \varphi(t)dt \leq r_i, \; i = 1,...,n,$$

then from (2.10)

$$\int_0^{d_i(u_i,w_i)} \varphi(t)dt \leq \lambda r_i, \; i = 1,...,n.$$

Inductively

$$\int_0^{d_i(u_i,w_i)} \varphi(t)dt \leq \lambda^m r_i$$

Implies that $u_i = w_i$, $i = 1, ..., n$, which completes the proof.

REMARK 2.1. In Theorem 2.1, if we take $n = 1$, $a_{ik} = a_{11} = k$ and $\beta = \max\{k, b\}$ then we have the Theorem 2 of Rhoades (2003) as a special case. Further if we take $\varphi(t) \equiv 1$we get theorem of Ciric (1971).

REMARK 2.2. In Theorem 2.1, if we take $b = 0$ and $\varphi(t) \equiv 1$ then it becomes the Matkowski's contraction (1975).

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