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BULK QUEUEING MODEL WITH VACATION AND NON-IDENTICAL SERVERS

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ABSTRACT

In the present paper we consider a queueing model in which customer arrives according Poisson pattern and bulk service with grand vacations process. We discuss stationary queue size distribution, departure point queue size distribution, queue size distribution at stationary points of vacation period and joint distribution of the queue size and vacation period.

KEY WORDS: Bulk queue, grand vacation, Poisson's process, non-identical servers.
MATHEMATICS SUBJECT CLASSIFICATION (2000): 60K25, 60K30.

INTRODUCTION

The queues with balking have been studied by many researchers. Finch (1959) studied GI/M/1 queueing with balking. Haight (1960) describe the queue with balking in details. Rao (1966) consider queueing model with balking reneging and limited servers. In (1968), Rao discussed queueing with balking, reneging in M/G/1 queueing system. Satyamutri (1971) develop a simple method to study the transient behaviour of the system. Borthakur (1975) discuss differential equations technique in a comprehensive manner which could easily be applied in balk service sequence for analyzing various performances as queue length, waiting time distribution etc.

Chaudhary and Baruh (1984) discuss M/M/1 queueing model with threshold policy and grand vacations process where the server takes a sequence of vacations till the return to find at least some pre-specified number of customers after each grand vacations. Haghighi et al. (1986) investigate multi-servers Markovion queueing system with balking and reneging. Minh (1988) obtain transient solution for some exhaustive M/G/1 queue with generalized independent vacations, while Kella (1989) discuss the optimal control of the vacation scheme in M/G/1 queueing model. Mishra et al. (2005) consider bulk service queue with vacation and non-identical servers.

In the present paper we consider bulk queueing model $M/G^k/3$ for non identical servers with fixed service and a grand vacations process. Various operating characteristics as stationary queue size distribution, departure point queue size distribution at the stationary point of vacation period and joint distribution of the queue size and vacation period has been discussed.

STATIONARY QUEUE SIZE DISTRIBUTION

We consider $M/G^k/3$ infinite capacity system with balk services. Let customers arrive in system according Poisson process with arrival rate λ. The arriving customers are served by three non-identical servers in batches of fixed size, say k, according to FCFS (first come first served) discipline. The three non-identical servers have different service rates μ_1, μ_2 and μ_3 respectively, which are independent and non-identically distributed and μ is average service rate.

Let $N_Q(t)$ be the number of customers in the system at time t. If after completion of the service, the servers find less than k customers, then they will go on vacations for random duration and after vacation, if servers find k or more customers waiting in queue, then they take a batch of k customers in order to their arrival for service while other wait. This generates vacation period independent and non-identically distributed random variable say V_1, V_2......... V_m respectively. Let V_i, V_j and V_k be the generic random variable of the sequence of vacation period for the first, second and third servers respectively where i = 1, 2......m_1, j = 1, 2...... m_2 and k = 1, 2......m_3.

Hence V_i, V_j and V_k are considered as grand vacations comprised of m_1, m_2 and m_3 vacations respectively. The probability of k or more arrivals during m_1, m_2 and m_3^{th} vacations are given by

$$p_{m_1} = \int_0^\infty \frac{e^{-\lambda x}(\lambda x)^{m_1}}{!m_1} . dV_i(x) \; ; m_1 = 0, 1, 2... \tag{1}$$

$$p_{m_2} = \int_0^\infty \frac{e^{-\lambda x}(\lambda x)^{m_2}}{!m_2} . dV_j(x) \; ; m_2 = 0, 1, 2... \tag{2}$$

$$p_{m_3} = \int_0^\infty \frac{e^{-\lambda x}(\lambda x)^{m_3}}{!m_3} . dV_k(x) \; ; m_3 = 0, 1, 2... \tag{3}$$

where $V_i(x)$, $V_j(x)$ and $V_k(x)$ are distribution functions of V_i, V_j and V_k respectively.
Let the indicator function is

$$I(t) = \begin{cases} 1, & \text{if one or two or all servers are busy} \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Clearly $\{N_Q(t), I(t)\}$ is a continuous time Markov process, where $V_i^0(t)$, $V_j^0(t)$ and $V_k^0(t)$ are elapsed times at time t.
Let there are n customers in the system with vacation periods. The system may have anyone of the following states:
$S_1 = \{n, 0, 0, 1\}$, $S_2 = \{n, 0, 1, 0\}$, $S_3 = \{n, 0, 1, 1\}$, $S_4 = \{n, 1, 0, 0\}$, $S_5 = \{n, 1, 0, 1\}$, $S_6 = \{n, 1, 1, 0\}$, $S_7 = \{n, 1, 1, 1\}$, $S_8 = \{n, 0, 0, 0\}$.

CONCLUSION:

In the present paper we have discussed the behaviour of queueing model $M/G_k/3$ with non-identical servers. In this model we also discussed the stationary queue size distribution, departure point queue size distribution at the stationary point of vacation period and the joint distribution of the queue size and a vacation period. Such types of models are wide useful in real life situation as flexible manufacturing, inventory system, transport system and computer system etc.

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