J. Mountain Res. Vol 2 pp.67-76, 2007

ON FIXED POINTS OF FUZZY MAPPINGS

U.C. Gairola* R.C. Dimri and V.B. Chandola****

Department of Mathematics*

H.N.B.Garhwal University Campus, PAURI 246001

E-mail: ucgairola@rediffmail.com

Department of Mathematics**

H.N.B.Garhwal University Srinagar Garhwal 246174

E-mail: dimrirc@gmail.com

ABSTRACT

In this paper some common fixed point theorems for a pair of fuzzy mappings in complete quasi-metric spaces are established. Our results Generalize and extend fixed point theorems of Gregori-Romaguera (2000) and Park-Jeonqc(1997).

AMS (MOS) Subject classifications (2000): 47H10, 54H25

Key Words and Phrases: Fuzzy mapping, quasi-metric spaces, fixed point.

INTRODUCTION

Heilpern(1981) first introduced the concept of fuzzy mappings and proved a fixed point theorem for fuzzy contraction mappings which is a fuzzy analogue of the fixed point theorem for multivalued mappings of Nadler (1969). Further Bose and Sahni(1987) have extended the result of Helpern[op cit.] for a pair of generalized fuzzy contraction mappings. The fixed point theory of fuzzy mappings in complete metric spaces has periodically received certain attention (see, for instance Arora-Sharma(2000), Bose- Sahni[op cit.],Butnariu(1982),Chang(1985,1988,1994), Chitra(1986), Lee and Cho(1994),Lee *et al.* (1998),Park-Jeong(1997) etc.). On the other hand, the theory of quasi-uniform and quasi- metric spaces has been considered and applied in the last few years by many authors, to the study of hyperspaces, function spaces, fuzzy topology, fixed point theory etc.(see, for instance Berthiaume(1977),Cao etal.(2000), Fletcher-Lendgren (1982), Gregori-Romaguera (2000), Jachymski (1993Kunziet al(1983), Kunze Romaguera (1997,1998),Romaguera (1992),Smyth (1987)and Sunderhauf (1994,1995)). In the light of these facts, it seems interesting to study the fixed point theory of fuzzy mappings in quasi-metric spaces. In this paper, we prove some common fixed point theorems for fuzzy mappings in complete quasi-metric spaces. Our results generalize and extend some fixed point theorems of Gregori-Romaguera(2000) and Park-Jeong(1997) for a pair of mappings.

2. NOTATIONS AND DEFINITIONS

A quasi-metric on a nonempty set X is a nonnegative real valued function d on $X \times X$ such that for all $x, y, z \in X$

(i) $d(x,y) = d(y,x) = 0 \Leftrightarrow x = y$ and

(ii) $d(x,y) \leq d(x,z) + d(z,y)$.

A quasi- metric space is a pair (X, d) such that X is a nonempty set and d is a quasi – metric on X. Each quasi-metric d on X generates a T_0 topology $\tau(d)$ on X. The function d^{-1}, defined on $X \times X$ by $d^{-1}(x, y) = d(y, x)$, is also a quasi-metric on X called the conjugate of d. The metric d^s defined by

$$d^s(x, y) = \max\{d(x, y), d(y,x)\},$$

for all $x, y \in X$.

Fleetcher–Lindgren(1982) and Kunzi(1995) provide two good references for quasi-metric spaces and related concepts.

A sequence $\{x_n\}$ $n \in N$ in a quasi- metric space (X, d) is called left K-Cauchy(see, Reilly et.al.(1992)) if for each $\varepsilon > 0$ there is an $n_\varepsilon \in N$ such that $d(x_n, x_m) < \varepsilon$ for all $m,n \in N$ such that $n_\varepsilon \leq n \leq m$. The quasi-metric space (X, d) is left K- complete (see, Reilly et al.(1992) and Romaguera(1992)), provided that every left K-Cauchy sequence in (X, d) is convergent with respect to the topology $\tau(d)$. (X, d) is smyth-complete (see, Smyth(1994)), provided that every left K- Cauchy sequence in (X,d) is convergent in the metric space (X, d^s) (see also Kunzi(1995)). Clearly, every Smyth –complete quasi-metric space is left K-complete. However, in general the converse implication does not hold.

$d(a, c) \leq D_1 (P(a), Q(z))$ and hence $d(a, c) = 0$.

By condition (ii) of Definition 2.2, a is a fixed point of P.

Similarly we can prove that a is also a fixed point of Q.

REMARK 3.1. If we put P = Q in Theorem 3.1 and in Theorem 3.2 we get Theorem 1 and Theorem 2 of (Gregori- Romaguera (2000)) respectively.

REMARK 3.2. If we replace Smyth-complete quasi-metric spaces by complete metric spaces in Theorem 3.1 and in Theorem 3.2, we get the results of (Park - Jeong (1978)).

REFERENCES

Arora, S.C. and Sharma, V. 2002, Fixed point theorems for fuzzy mappings. *Fuzzy Sets & Systems,* 110, 127-130.

Berthiaume, G.1977, On quasi uniformities in hyper spaces, *Proc. Amer. Math. Soc.* 66, 335-343.

Bose, R.K. and Sahni, D. 1987, Fuzzy mappings and fixed point theorems. *Fuzzy Sets & Systems*, 21, 53-58.

Butnariu, D. 1982, Fixed points for. fuzzy mappings. *Fuzzy Sets & Systems*, 7, 191-207.

Cao, J. Kunzi, H.P.A., Reilly, I.L. and Romaquera, S., 2000, Fixed point theorems for fuzzy mappings in quasi-metric spaces. *Fuzzy Sets & Systems*, 115, 477-483.

Chang, S. -S., 1985, Fixed point theorems for fuzzy mappings. *Fuzzy Sets & Systems*, 17, 181-187.

Chang, S. -S., 1988, Coincidence degree and coincidence theorems for fuzzy mappings, *Fuzzy Sets & Systems*, 27, 327-334.

Chang, S. -S., 1994, Coincidence theorems and variational inequalities for fuzzy mappings. *Fuzzy Sets & Systems,* 61, 359-368.

Chitra, A. 1986, A note on the fixed points of fuzzy maps on partially ordered topological spaces. *Fuzzy Sets & Systems*, 19, 305-308.

Fletcher, P. and Lindgren, W.F., 1982, Quasi-uniform spaces, Marcel Dekker, New York.

Gregori, V. and Romaguere, S., 2000, Fixed point theorems for fuzzy mappings in quasi-metric spaces. *Fuzzy Sets & Systems,* 115, 477-483.

Heilpern, S. 1981, Fuzzy mappings and fixed point theorem. *J. Math. Anal. Appl.* 83, 506-509.

Jachymski, J. 1993; A contribution to fixed point theory in quasi-metric spaces. *Publ. Math. Debrecen* 43, 283-288.

Kunzi, H.P.A., 1995, Non symmetric topology. *Bolyai Soc. Math. Studies Hungary, Budapest* 4, 303-338.

Kunzi, H.P.A., Mrsevic, M., Reilly, I.L. and Vamanamurthy, M.K. 1983, Convergence, precompactness and symmetry in quasi-uniform spaces, *Math. Japonica*, 39, 239-253.

Kunzi, H.P.A., and Romaguera, S., 1997, Space of continuous functions and quasi-uniform convergence, *Acta. Math. Hungar.* 75, 287-298.

Kunzi, H.P.A., and Romaguera, S., 1998, Well quasi-or dering and the Hausdorff quasi-uniformity. *Topology Appl.* 85, 207-218.

Lee, B.S. and Cho, S.J., 1984, A fixed point theorem for contractine type fuzzy mappings. *Fuzzy Sets & Systems,* 61, 309-312.

Lee, B.S. Lee, G.M., Cho, S.J. and Kim, D.S. 1998, A common fixed point theorem for a pair of fuzzy mappings, *Fuzzy Sets & Systems*, 98, 113-136.

Nadler, Jr., S.B., 1969, Multivalued contraction mappings, *Pacific J. Math.* 30, 475-488.

Park, S. 1978, Fixed points of f-contractive maps, *Rocky Mountain J. Math.* 8,743-750.

Park, J.Y. and Jeong, J.U., 1997, Fixed point theorems for fuzzy mappings. *Fuzzy Sets & Systems* 87, 111-116.

Reilly, I.L., Subrahmanyam, P.V., and Vamanamurthy, M.K.,1992, Cauchy sequences in quasi-pseudometric spaces. *Monatsh, Math. Nachr.* 157, 15-23.

Romaguera, S. 1992, Left k-completeness in quasi-metric spaces, *Math. Nachr.* 157, 15-23.

Smyth, M.B., 1987, Quasi-uniformities: Reconciling domains and metric spaces, Lecturer Notes in Computer Science 298, (Springer, Berlin-New York), 236-253.

Smyth, M.B., 1994, Completeness of quasi-uniform and syntopological spaces. *J. London Math. Soc.* 49(2), 385-400.

Sunderhauf, P. 1994, The symyth-completion of a quasi-uniform and syntological spaces. *J. London Math.* Soc. 49(2), 385-400.

Sunderhauf, P. 1995, Constructing a quasi-uniform function space, *Topology Appl.* 67, 1-27.