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SOME FIXED POINT THEOREMS FOR EXPANSION TYPE MAPPINGS IN MENGER SPACES

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ABSTRACT

Fixed point theorems for a pair as well as a sequence of expansion type mappings in a Menger space have been established. These results are extension and generalization of some well-known results of Gillespie-Williams, Taniguchi, Rhoades, Wang-Li-Gao-Iseki and Pant-Dimri-Singh in metric and Menger spaces.

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INTRODUCTION

A comparison of definitions of contractive type mappings on a complete metric space has been discussed by B.E. Rhoades (1977) and obtained some general theorems on fixed points. Wang *et.al.,* (1984) introduced the concept of expansion mappings which correspond some contractive mappings in Rhoades (1977) and considered the existence of the fixed points on metric spaces. Further generalizations of these results have been obtained by Gillespie and Williams (1983), Popa (1986-87) and Taniguchi (1989) and Rhoades (1985). Recently a number of authors have proved some fixed point theorems for expansion type mappings in metric spaces (Taniguchi, 1989;Daffer and Kaneko, 1992; Hicks and Seliga, 1993; Jachymski, 1995).

The notion of expansion mappings on a probabilistic setting has been introduced by Pant *et.al.,*(1987) together with some fixed point theorems or such mappings. Further generalization of these results on Menger spaces have been obtained by Vasuki (1991). In this paper we generalize some of the results of Rhoades (1985) and Wang *et.al.,* (1984) in Menger spaces for a pair and a sequence of expansion type mappings.

PRELIMINARIES

Definition 2.1: Let X be a non-empty set and f a mapping from XxX to L,the collection of all distribution functions. An ordered pair (X, f) is called a probabilistic metric space(PM space) if it satisfies the following conditions in which $F_{u,v}$ denotes the value of f at $(u,v) \in X \times X$:

(a)$F_{u,v}(x) = 1$ for all x>0 iff u=v;

(b)$F_{u,v}(0)=0$;

(c)$F_{u,v}(x)=F_{u,v}(x)$;

(d)If $F_{u,v}(x)=1$ and $F_{v,w}(y)=1$ then $F_{u,w}(x+y)=1$.

For all u, v, w $\in$ X and all x, y $\geq$ 0.

Definition 2.2: A Menger space is a *triplet(X, f,t) consisting of a PM-spaces (X, f) and the t-norm* (Schweizer and Sktar, 1983) such that t is replaced by

(d') $F_{u,w}(x + y) \geq t\{F_{u,v}(x), F_{v,w}(y)\}$

for all u, v, w $\in$ X and all x, y $\geq$ 0.

Note that among a number of possible choice for t, t(a, b) =min{a, b} or simply " t = min" is the strongest possible universal t (Schweizer and Sktar, 1983, p. 318).

For details of topological preliminaries on Menger spaces, we refer to Schweizer and Sklar (Op. Cit.)

If $x \in X$, $\varepsilon > 0$ and $l \in (0,1)$, then an (ε, λ)-neighbourhood of x, called $U_x(, \lambda)$, is defined by

$U_x(\varepsilon, \lambda) = \{ y \in X: F_{x,y}(\varepsilon) > 1-\lambda\}$

If t is continuous, then (X,f,t) is a Hausdorff space in the topology induced by the family $\{U_x(\varepsilon,) : x \in X, \varepsilon>0, \lambda \in (0,1)\}$ of neighbourhoods.

In the (ε,l)-topology a sequence $\{x_n\}$ in X converges to an element $x \in X$ iff for every ε>0 and λ>0 there exists an integer N(ε,λ) such that

$F_{x_n, x}(\varepsilon) > 1 - \lambda$ whenever $n \geq N(\varepsilon, \lambda)$.

Sehgal and Ried (1972) have obtained the following lemma, which is useful to prove our theorems.

Lemma: 2.3: Let {Xn} be a sequence in a Menger space (X, f, t) with t continuous and $t(x, x) \geq x$, for every $x \in [0,1]$. Then $\{x_n\}$ is Cauchy iff $F_{x_{n+1}, x_n}(\varepsilon h) \geq F_{x_n, x_{n-1}}(\varepsilon)$, n =1,2,3......,for all ε >0 and 0<h<1.

Definition 2.4[7]: Let X be a Menger space. A mapping $f : X \to X$ is called an expansion mapping iff for a constant h > 1

$F_{fu, fv}(hx) \leq F_{u, v}(x)$

for all u, v in X and x $\geq$ 0.

$f(u,v) = F_{u,v}(x) = H(x-d(u,v))$ where $H(x) = 0$ for $x \leq 0$ and $H(x)=1$ otherwise.

If t-norm t is $t(a, b) = \max \{a,b\}$, then clearly (M,f,t) is a complete Menger space. The rest of the proof follows from corollary 3.3 of Pant *et.al.*(1987).

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