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Electric Quadrupole Moment of ^{7}Li nucleus using Complex Generator Co-ordinate Technique

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Abstract: The electric quadrupole moment (EQM) of a nucleus is an important parameter used to determine the departure of the nucleus from its spherical shape. Besides the study of the nucleus, determination of the EQM value finds applications in solid state physics and in other areas such as chemical, medical and biological sciences. This paper focuses on a method developed for the calculation of nuclear structural properties of light nuclear systems which is used here to calculate the nuclear electric quadrupole moment for the ground state of 7lithium nucleus. In this approach the required anti-symmetrized nuclear wave function is constructed by using combination of shell model, cluster model, resonating group method, generator coordinate method, and the complex generator coordinate technique. The ground state of 7li nucleus is considered as a nuclear system consisting of one alpha cluster, one deuteron cluster and a neutron cluster. The wave function is written by taking internal spatial function, spin, and iso-spin functions where arguments of the internal function of clusters include the parameter coordinates and generator coordinates. The consideration of the parameters viz., total angular momentum, spin, and definite parity of the nucleus makes the approach inclusive. While the relative motion wave functions are taken into account between the alpha cluster and deuteron cluster and secondly between the alpha cluster and neutron cluster. The potential chosen in this approach is the Wood-Saxon potential which is somewhat between harmonic oscillator potential well and square well potential. By incorporating all the aforesaid inputs, a matrix of 7×7 order is constructed which leads to the calculation of EQM of 7lithium. The results compared with the available experimental and other theoretical data. The present value of EQM is found in good agreement with experimental data.

Keywords: Quadrupole moment • Cluster model wave function • resonating group method • complex generator coordinate technique

Introduction

The nuclear electric quadrupole moment (EQM) measures the deviation of charge distribution from a spherical shape and thus provides information about nuclear shape (Schiller and Schmidth 1935). The measurement of EQM involves interaction of the nuclear charge distribution with the static charge distributions of electrons in atomic and molecular systems, or with a specified external applied electric field.

There are various methods for determining nuclear shape, but we have focused on the interaction of nuclei with fast charged particles, i.e., electron scattering (Meyer et al 1959). In electron scattering experiments, the measurement of scattering cross section is interpreted in terms of a charge distribution over a finite radius.

Formulation of Electric Quadrupole moment

The atomic nuclei can be considered as a small electric charge and magnetic moment distributions (Xiao et al 2018) (James, 1998). The electrostatic energy of a localized charge distribution described by charge density ρ(x) placed in an external potential (Jackson, 1962) $\Phi(\mathrm{x})$ is

$$W = \int \rho(x)\Phi(x)dx^3 \quad (1)$$

Since the nucleus is of a small dimension the potential $\Phi(\mathrm{x})$ will change by very small amount over the nuclear volume and $\Phi(\mathrm{x})$ can be explained in a Taylor's series around a suitably chosen origin

$$\Phi(x) = \Phi(0) + \bar{x}.\nabla\Phi(0) + \frac{1}{2}\sum_{ij} x_i . x_j . \frac{\partial^2\Phi(0)}{\partial x_i \partial x_j} + -- \quad (2)$$

Since the electric field E = $-\nabla\Phi$,substituting in above equation and for the external field, one can subtract $\frac{1}{6}r^2\,\nabla.E\,(0)$from eq. (2)

$$\Phi(\mathrm{x}) = \Phi(0) - \bar{\mathrm{X}}.\bar{\mathrm{E}}(0) - \frac{1}{6}\sum_{ij}\left(3x_ix_j - r^2\delta_{ij}\right)\frac{\partial E_j(0)}{\partial x_j} \quad (3)$$

In this equation the first term represents the potential at x if the total charge were concentrated at the origin and is called the potential due to monopole moment of charge distribution. The second term represents the potential which would result if a point dipole having dipole moment equal to that of the charge distribution within the nucleus. The third term is called the potential due to quadrupole moment of charge distribution and other higher order terms are called multipole potentials. The Quadrupole moment tensor Q_{ij} is given by

$$Q_{ij} = \int \left(3x_ix_j - r^2\delta_{ij}\right)\rho\,(x)d^3x \quad (4)$$

Since the wavefunction of a stationary state has a definite parity, because of symmetry inversion for P(x) the nuclear electric dipole moments are essentially zero. The charge density of the nucleus is cylindrically symmetric about the z-axis. Thus, the only non-vanishing QM tensor component is Q_{33}**.**

The quantum mechanical expressions for the operators corresponding is total charge, dipole moment and QM are obtained by substituting the following

$$\int \rho(r)d^3r = \int Z\,e\,|\Psi\,(r_1r_2 ------ r_A)|^2 d^3r_1d^3r_2 --- \; d^3r_A \quad (5)$$

where, **Ψ** (r_1r_2 ----- r_A) is the nuclear wavefunction, r_1 r_2 ----- r_A are position coordinates of A nucleons in the nucleus.

The expression for quadrupole moment

$$Q_{ij} = \sum_{K=1}^{Z} e\left(\left(3x_ix_j - \delta_{ij}r^2\right)_K |\Psi\,(\bar{r_1} ----- \bar{r}_A)|^2 d^3r_1 --- \; d^3r_A \quad (6)$$

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From eq. (6) the quadrupole moment is the expectation value of the operator $\frac{1}{e}(Q_{33})$ in the nuclear state Φ_{JM} with M = J where e is the proton charge.

$$Q_{JM} = \frac{\frac{1}{e}\langle \Psi_{J,M} |\int (3z^2 - r^2)\, P(\bar{r})\, d^3r | \Psi_{JM}\rangle}{\langle \Psi_{JM} | \Psi_{JM}\rangle} \qquad (7)$$

The unit of quadrupole moment is length^2.

Calculation of Electric Quadrupole Moment

For the calculation of electric quadrupole moment of ^7Li nucleus in ground state first we have written the cluster shell model wavefunction (Sinha and Sinha 2021) by assuming that it is made of one alpha (2p and 2n) cluster, one deuteron (1p and 1n) cluster and neutron cluster due to long range correlation between protons and neutrons. The spin and parity of the nucleus in ground state is $J^\Pi = 3/2^-$. The shell model starts from an average potential with a shape something between square well and harmonic oscillator (Tang and Wildermuth 1977). In this case we have used a more realistic but at the same time a complicated potential known as Wood-Saxon potential. The integral representation for the wavefunction Φ for the ground state of 7Li nucleus is

$$\phi_{\frac{3}{2}\frac{3}{2}} = A \int \prod_{J=1}^{4} \prod_{K=5}^{6} \xi_\alpha \xi_d \xi_n \exp\left\{\frac{-\alpha}{2}(\bar{r}_j - i\bar{p})^2\right\}$$

$$\exp.\left\{\frac{-\alpha}{2}(r_K - iQ)^2\right\} \exp.\left\{\frac{-\alpha}{2}(\bar{r}_7 + 4i\bar{p} + 2iQ)^2\right\}$$

$$\chi_{11}(\bar{R}_1'')\chi_{10}(\bar{R}_2'') \ \exp.\left[-2\alpha\{(\bar{P}+\bar{Q})^2 - 2\bar{P}.\bar{Q}\} + \right.$$

$$\alpha R_1''^2 + \frac{4}{7}\alpha R_2''^2 - 2i\,\alpha\bar{R}_1''.(\bar{P}-\bar{Q}) - \frac{36i}{7}\alpha R_2''.(\bar{P}+\bar{Q}) -$$

$$8\alpha(\bar{P}+\bar{Q})^2\, d\bar{P}\, d\bar{Q}\, dR_1''\, dR_2'' \qquad (8)$$

substituting eq. (8) in eq. (7) and solving by fitting the value of width parameters of alpha and deuteron clusters, the calculated value of EQM is – 0.03881 barns. Here negative sign shows oblate shape of nucleus.

Results and Discussion

In the present calculations the value of quadrupole moment has been calculated as-0. 03881 barns. We have fitted the value of alpha and beta parameters from the binding energy calculations for the ground state of ^7Li nucleus (Sinha and Sinha 2022). The present value of EQM is compared with the theoretical calculation which is - 0.03992 barns and – 0.03978 barns (Doma et al 2020) and with one more theoretical calculation is – 0.03935 barns (Sharif 2016). Our results are in good agreement with the experimental value where EQM is 0-.0408 barns (Raghavan 1989) and one more theoretical value -.044 barns (Kahalas 1963). The technique of writing wavefunction is already used for the calculating different parameters for ^{5}He and ^{6}Li nucleus (Sinha and Sinha 2020) in the ground state. The wavefunction was employed to calculate the charge form factor for ^{5}He nucleus (Sinha and Sinha 2019) agreement with the experimental data. We can further use the presently developed wavefunction to calculate charge form factor for ^{7}Li as well.

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